Determination of Stabilizing Parameter of Fractional Order PID Controller Using Genetic Algorithm

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Abstract

This paper present the development of new tuning method and performance for Genetic Algorithm based Fractional order PID controller ,fractional order PID controller include the conventional order PID controller parameter. The tuning of the PID controller is mostly done using Zeigler and Nichols tuning method. All the parameters of the controller, namely K_p (Proportional gain), K_i (integral gain), K_d (derivative gain) can be determined. Fractional order PID (FOPID) is a special kind of PID controller whose derivative and integral order are fractional rather than integer. To design FOPID controller is to determine the two important parameters λ (integrator order) and μ (derivative order).The key challenge of designing of FOPID to determine the values of λ and μ . Many random search methods, such as genetic algorithm (G.A) have recently received much interest for achieving high efficiency and searching global optimal solution in space. Due to the high potential for global optimization, GA has received great attention in control system to determine the optimal FOPID controller parameter. This paper proposed a tuning method of FOPID by using Genetic Algorithm.

Keywords: PID, Fractional order PID, Genetic Algorithm

I. INTRODUCTION

PID controller provides feedback, it has ability to eliminate steady state offsets through derivative action. The derivative action in the control loop will improve the damping and therefore by accelerating the transient response, a lighter proportional gain can be obtained during the past half century, many theoretical and industrial studies have been done in PID controller setting

rules Zeigler and Nichol's in 1942 proposed a method to set the PID controller parameter Hagglund and Astrom in 1955 and cheng-ching in 1999, introduced other technique. By generalizing the derivative and integer orders, from the integer field to non-integer numbers, the fractional order PID control is obtained.

The performance of the PID controller can be improved by making the use of fractional order derivatives and integrals. This flexibility helps the design more robust system. Before using the fractional order controller in design an introduction to the fractional calculus is required. Recently, fraction order calculus and its application have been attracting more and more interest. In many fields, fractional order controllers have demonstrated their advantages. Many researchers have focused on fractional order PID controllers. Today, a central issue in the field of control systems is effect of uncertainty parameter.

Fractional order systems could model various real materials more adequately than integer order ones and thus provide an excellent modeling tool in describing many actual dynamical processes. It is intuitively true that these fractional order models require the corresponding fractional order controllers to achieve excellent performance. In general, there is no systematic way of designing proper fractional order controller for the fractional order system.

Genetic Algorithm is a search technique that manipulates the coding representation of a parameter set to search a near optimal solution through cooperation and competition among the potential solutions. This algorithm is highly relevant for the industrial application, because this algorithm is capable of handling problem with constraints, objectives and dynamic components. GA uses such natural evolution to get the global optimization

1. Conventional Order PID Controller

Three term or PID controllers are probably by most widely used industrial controller. A PID controller is essentially a generic closed loop feedback mechanism.

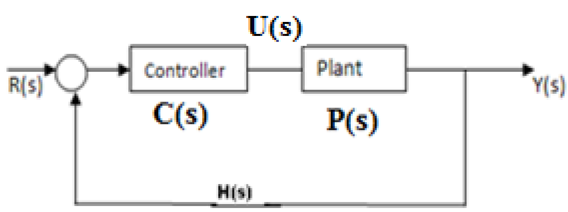

Fig.1 Unity feedback control system

In working principle is that it monitors the error between a measured process variable and a desired set point; from this error, a corrective signal is computed and is eventually feedback to the input side to adjust the process accordingly. The differential equation for the PID controller is

$$u(t) = K_p e(t) + T_i \int_0^t e_p(t)dt + T_d \frac{d}{dt} e(t)dt \quad (1)$$

Thus, the PID controller algorithm is described by a weighted sum of the three times functions were the three distinct weights are: K_p (Proportional gain) determines the influence of the present error value on the control mechanism, I (integral gain) decides the reaction based on the area under the error time curve up to the present point and Td (derivative gain) accounts for the extent of the reaction to the rate of change of the error with time.

1(a).Tuning of Integer Order PID Controller

Many tuning method are presented in literature that are based on a few structure of the process dynamics.

1.1(a) Zeigler Nichols Process Reaction Curve Method

In 1942, Zeigler and Nichols presented two classical methods to tune a PID controller. These methods are widely used, due to their simplicity. In the first method, the controller setting are based on two parameter θ and a of the process reaction curve.

1.1(c) Astrom and Haggland (1985)

Astrom and Haggland recognized that the Zeigler-Nichols continuous cycling method actually identifies the point ($-1/K_u$, 0) on the Nyquist curve, and move it to a predefined point. With PID control, it is possible to move a given point on the Nyquist curve to an arbitrary position. By increasing the gain, the arbitrary point moves in the direction of $G(j\omega)$. By changing I and D action moves the point in the orthogonal direction.

2. Brief Mathematical Background of Fractional calculus

Orders of fractional calculus are real number. Many different definitions for general integro- differentional operation can be found in the literature. Among them the most commonly used for general fractional Integro-differential expressions are given by chauchy, Riemann-Liouville, Grunwald letnikov and Caputo. These definitions are required for realization of control algorithm.
At first, we generalize the differential and Integral operators in to one fundamental operator ${}_aD_t^\alpha$ where

$$
\begin{aligned}
{}_aD_t^\alpha &= \frac{d^\alpha}{dt^\alpha} \quad for \quad R(\alpha) > 0 \\
&= 1 \quad for \quad R(\alpha) = 0 \\
&= \int_a^t (d\tau)^{-\alpha} \quad for \quad R(\alpha) < 0
\end{aligned}
$$

$R(\alpha)$ Denote real part of the α which is, in general is a complex quantity.
The Grunwald – Letnikov definition is

$$ {}_aD_t^{\alpha} f(t) = Lt_{h\to 0} \frac{1}{h^{\alpha}} \sum_{j=0}^{\left[\frac{t-a}{h}\right]} (-1)^j \binom{\alpha}{j} f(t-jh) $$

2.1 Basic concept of Fractional order PID controller

Consider the negative feedback control system as shown in fig (1).The continuous transfer function of the $PI^{\lambda}D^{\mu}$ controller is obtained through Laplace transform as

$$ C(s) = K_p + \frac{T_i}{s^{\lambda}} + T_d s^{\mu} \qquad (2) $$

The PID controller expands the integer order PID controller from point to plane, there by adding flexibility to controller design and allowing us to control our real world processes more accurately but only at the cost of increased design complexity.

3. Design Specification

A fractional order controller system fulfills different specifications regarding to the plant uncertainties, load disturbance and high frequency noise [14]. Therefore, the design problem is formulated as follows.

i) Phase Margin and Gain crossover frequency specification
Gain and phase margin have always served important parameter for robustness. It is known that the phase margin is related to the damping of the system. The equations that define the phase margin ϕ_{pm} and gain crossover frequency ω_{cp} are

$$ \left|G(j\omega_{cp})\right| = \left|C(j\omega_{cp})P(j\omega_{cp})\right|_{dB} = 0dB \qquad (3) $$

$$ \text{Arg}[G(j\omega_{cp})] = \text{Arg}[C(j\omega_{cp})P(j\omega_{cp})] = -\pi + \phi_{pm} \qquad (4) $$

ii) Robustness to gain variation in the gain of the plant
The gain variation of the plant demands that the phase directives w.r.t the frequency is zero, i.e. the phase bode plot is flat, at the gain crossover frequency.

$$ \left(\frac{d(\text{Arg}(G(j\omega))}{d\omega}\right)_{\omega=\omega_{cp}} = 0 \qquad (5) $$

4. Design of IOPID and FOPID controller

In this paper, we focus on design of IOPID and FOPID controller. The transfer function for the controller are defined as

For IOPID controller $C_1 = K_p + \frac{K_i}{s} + sK_d \qquad (6)$

For FOPID controller $C_2 = K_p(1 + \frac{K_i}{s^{\lambda}} + s^{\mu}K_d) \qquad (7)$

As we known, the phase and gain of the integer order PID controller are mainly decided by K_p , K_i and K_d . However, the non-integer order controller is more flexible because the order λ and μ is added as an adjustable parameter.

4.1) Integer order PID controller Design

The open loop transfer function $G_1(s)$ for the IOPID with plant system is given as

$$ G_1(s) = C_1(s)P(s) \qquad (8) $$

According to the IOPID controller transfer function (6), we can get the frequency response as,

$$ C_1(j\omega) = K_p + \frac{K_i}{j\omega} + j\omega K_d $$

The gain and phase are as follow,

$$ \left|C_1(j\omega)\right| = \sqrt{K_p^{\ 2} + (K_d\omega - (K_i/(\omega K_p))^2} \qquad (9) $$

$$ Arg[C_1(j\omega)] = \tan^{-1}((K_d\omega^2 - K_i)/(\omega K_p)) \qquad (10) $$

Then the open loop frequency response

$$ G_1(j\omega) = C_1(j\omega)P(j\omega) $$

The gain and phase of the open loop frequency response are as follows by using (4) and (5)
According to specification (i), the phase of $G_1(j\omega)$ can be expressed and according to specification (ii) about the robustness to gain variation in the plant,

$$ \left(\frac{d(A\text{rg}(G_1(j\omega))}{d\omega}\right)_{\omega=\omega_{cp}} = 0 $$

According to the specification (ii), we established an equation about K_p,

$$ \left|G_1(j\omega_{cp})\right| = \left|C_1(j\omega_{cp})\right|\left|P(j\omega_{cp})\right| $$

From required equation by specification (i) and (ii) , we can get the parameter K_p, K_i and K_d if the set of the parameter(ω_c and ϕ_{pm}) get defined .

4.2) Fractional order PID controller Design

The open loop transfer function $G_2(s)$ of the fractional order PID controller for plant system is given by

$$G_2(s) = C_2(s)P(s)$$

According to the fractional order PID controller transfer function given as

$$C_4 = K_p\left(1 + \frac{K_i}{s^\lambda} + s^\mu K_d\right)$$

The frequency response of FOPID controller as

$$C_2(j\omega) = K_p(1 + K_i(j\omega)^{-\lambda} + K_d(j\omega)^\mu)$$

$$=K_p(1+[K_i\omega^{-\lambda}\cos(\frac{\lambda\pi}{2})+K_d\omega^{\mu}\cos(\frac{\mu\pi}{2})]+j[K_d\omega^{\mu}\sin(\frac{\mu\pi}{2})-K_i\omega^{-\lambda}\sin(\frac{\lambda\pi}{2})])$$

The gain and phase are as follow

$$|C_2(j\omega)| = K_p\sqrt{\left[1+K_i\omega^{-\lambda}\cos(\frac{\lambda\pi}{2})+K_d\omega^{\mu}\cos(\frac{\mu\pi}{2})\right]^2+\left[K_d\omega^{\mu}\sin(\frac{\mu\pi}{2})-K_i\omega^{-\lambda}\sin(\frac{\lambda\pi}{2})\right]^2} \quad (11)$$

And

$$Arg[C_2(j\omega)] = \tan^{-1}\left(\frac{K_d\omega^{\mu}\sin(\frac{\mu\pi}{2})-K_i\omega^{-\lambda}\sin(\frac{\lambda\pi}{2})}{1+K_i\omega^{-\lambda}\cos(\frac{\lambda\pi}{2})+K_d\omega^{\mu}\cos(\frac{\mu\pi}{2})}\right) \quad (12)$$

According to the gain and phase of the plant in $|P(j\omega)|$ and $Arg(P(j\omega))$, the open loop frequency response $G_2(j\omega)$ is that

$$|G_2(j\omega)| = |C_2(j\omega)||P(j\omega)| \quad (13)$$

And

$$Arg(G_2(j\omega)) = Arg[C_2(j\omega)] + Arg[P(j\omega)] \quad (14)$$

According to the specification (i), the gain of $G_2(j\omega)$ an established an equation about K_p

$$|G_2(j\omega_{cp})| = |C_2(j\omega_{cp})||P(j\omega_{cp})| = 1 \quad (15)$$

And the phase of $G_2(j\omega)$ can be determined by specification (ii).

Then relationship between K_i , K_d , λ and μ can established

According to the specification (ii) about the robustness to gain variation in plant

$$\left(\frac{d(Arg(G_4(j\omega))}{d\omega}\right)_{\omega=\omega_{cp}} = 0$$

By differentiate, we can established another equation about K_i, K_d, λ and μ.

Clearly, we can solve equation.

The design procedure of fractional order PID controller is summarized

A. Given the value of gain cross over frequency(ω_{cp}) , and desired phase margin(ϕ_{pm})

B. Plot a curve , K_d w.r.t μ as per the equation 13

C. Plot a curve , K_i w.r.t μ as per the equation 14

D. Plot a curve , K_i w.r.t λ as per the equation 13

E. Plot a curve , K_d w.r.t λ as per the equation 14

F. Obtain the values of K_d and μ from the intersection point of above two graph

G. Calculate the value of K_p from equation 11

5. Genetic Algorithm Based Tuning of FOPID Controller

The optimal value of the FOPID controller parameter λ, μ, is to be found using Genetic Algoritm.All Possible set of controller parameter value are particles whose value are adjusted to minimize the objective function. For the FOPID controller design, the controller settings estimated in a stable closed loop system.

To start with Genetic Algorithm, certain parameter needs to be defined. These include population size, bit, length of chromosomes, number of iterations, selection, crossover and mutation types. These selected parameters decide the ability of the designed controller. The basic flowchart of implementation of genetic algorithm shown in fig (3).

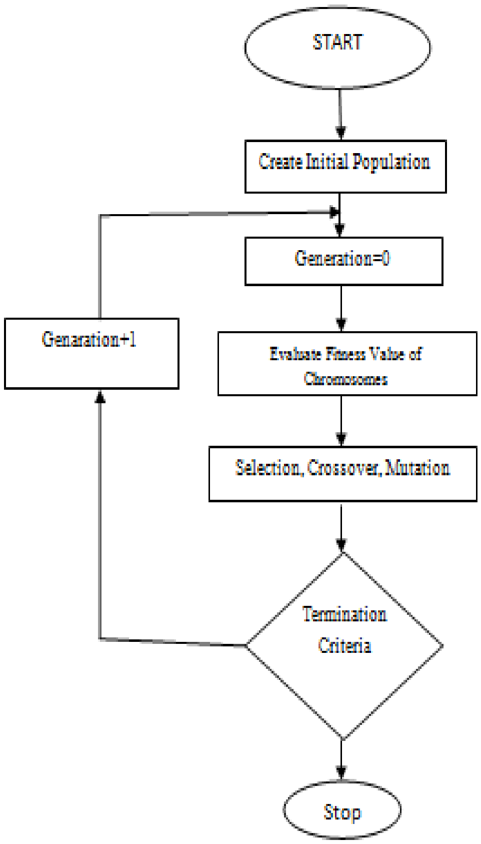

Fig.3 Basic Flow chart of Genetic Algorithm

The good set of values of λ and μ are considered as parents and to get the new set of values of λ and μ. If λ and μ are changed, its affects the unit step response of the system because λ and μ represents the integral and derivative term. The detail flow chart of finding out the values of λ and μ of fractional order PID controller using Genetic Algorithm.

To get the optimal value of the λ and μ by using genetic algorithms first initialize the values as follows:

Population type: Double vector

Population size: 100

Bit length of chromosomes: 6

Number of generations: 100

Mutation fraction: 0.01

Mutation Type: Uniform Mutation

Stopping criteria: ISE

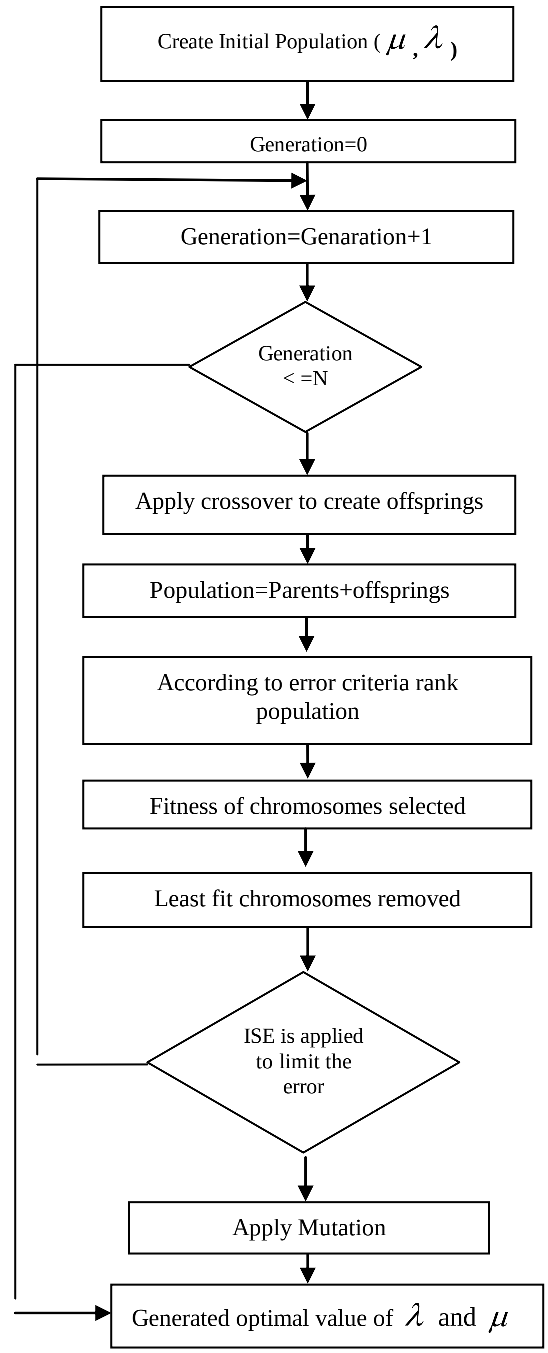

Fig.4 Detail flow chart to determine optimal value of λ and μ

To find the optimal value of λ and μ , a genetic Algorithm is used as shown in Fig(4).As new individuals are generally created as offspring of two parents. One and more so called cross over point are selected within the chromosomes of each parent. The good chromosomes are kept and the worst chromosomes are left out. Then ISE value of good chromosomes are checked to find out if these are the best chromosomes or optimal chromosomes, if the ISE value are not satisfied , then again crossover between the best chromosomes are performed till the ISE value comes within the limit. To get more optimal results, mutation operation is performed. By above steps the optimal value of λ and μ are achieved.

Simulation and Testing:

The transfer function considered for the implementation of PID and FOPID controller is given as

$$G(s)=\frac{20}{s^3+5s^2+41s+10}$$

The step response of the system using Integer order PID controller shown in figure(5) .In Integer order PID controller , the value of λ and μ are unity.

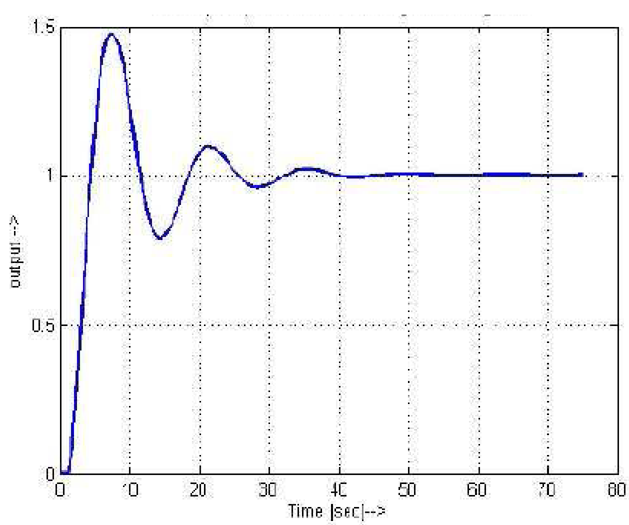

Fig.5.Unit Step Response of PID

		Ts	Mp	ISE
1	λ=1, μ=1	23.65	47.15	2.35

Table 1Time domain parameter of PID controller

The above response of controller with the parameter tuned using Zeigler-Nichols closed loop oscillation based tuning method. Table 1 shows the transient response of PID controller.

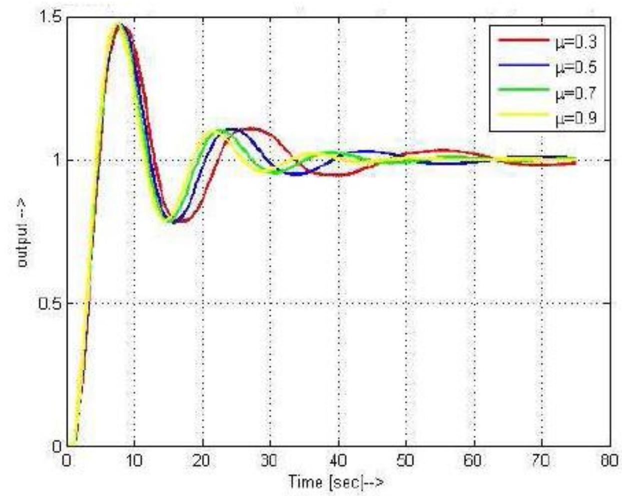

Fig. 6 Unit Step Response of FOPID with μ <1

		Ts	Mp	ISE
1	μ=0.3	41.742	46.0245	2.889
2	μ=0.5	34.903	46.9016	2.659
3	μ=0.7	25.293	47.2256	2.505
4	μ=0.9	24.148	47.3067	2.405

Table 2 Time domain parameter of FOPID controller with μ <1

The step response of the system using Fractional Order PID controller, where the derivative order μ and integral order λ of the fractional order PID system. In this integer order λ kept constant where as derivative order μ is changed. Here the derivative order is always less than 1.

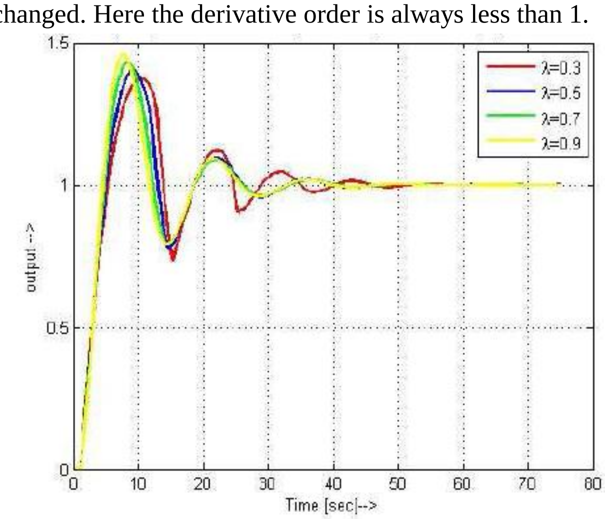

Fig.7 Unit Step Response of FOPID with λ <1

		Ts	Mp	ISE
1	λ=0.3	27.228	37.3379	2.243
2	λ=0.5	24.263	40.0857	2.253
3	λ=0.7	24.031	42.9715	2.291
4	λ=0.9	23.632	45.8391	2.335

Table 3 Time domain parameter of FOPID controller with λ <1

The step response of the system using Fractional Order PID controller, where the derivative order μ and integral order λ of the fractional order PID system. In this derivative order μ kept constant where as integral order λ is changed. Here the integral order is always less than 1. Table 3 shows the transient response of FOPID controller with λ <1.

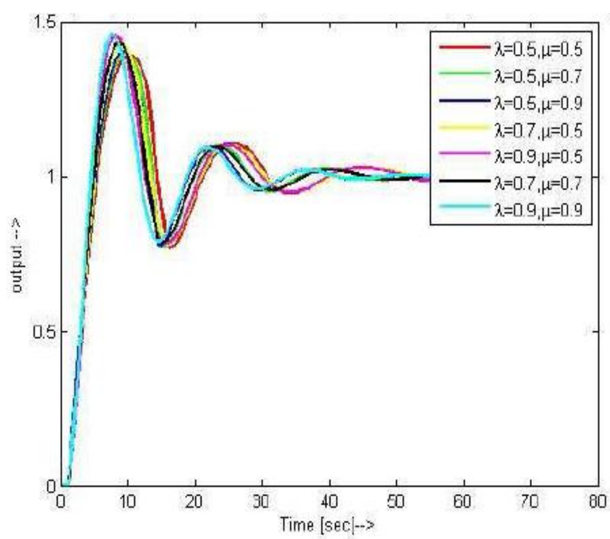

Fig.8. Unit Step Response of FOPID with λ <1and μ <1

		Ts	Mp	ISE
1	λ=0.5, μ=0.5	35.404	38.7205	2.543
2	λ=0.5, μ=0.7	26.266	39.7392	2.397
3	λ=0.5, μ=0.9	24.703	40.1304	2.300
4	λ=0.7, μ=0.5	28.661	42.1173	2.589
5	λ=0.9, μ=0.5	34.542	45.3238	2.634

Table 4 Time domain parameter of FOPID controller with λ <1and μ <1

The step response of the system using Fractional Order PID controller, where the derivative order μ and integral order λ of the fractional order PID system. In this integer order λ and derivative order μ is changed. Here the integral order and derivative order is always less than 1. Table 3 shows the transient response of FOPID controller with λ <1and μ <1.

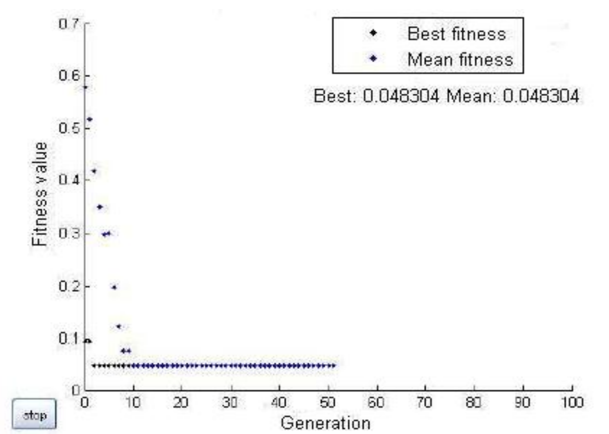

Fig.9 Fitness and Generic value graph of genetic algorithm

To find the optimal value of the derivative order μ and integral order value λ. This paper use genetic algorithm. Figure (9) shows the graph between the generation and fitness values.

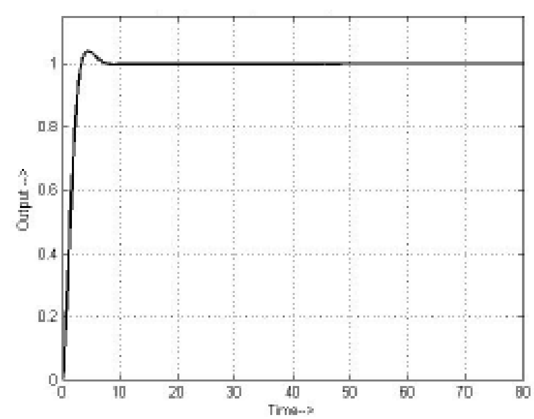

Fig.10 Unit Step response of FOPID by using optimal value of λ and μ

The step response of the system using fractional order PID controller with optimally tuned value of λ and μ . The optimal value of and are generated using genetic algorithm.

6. CONCLUSION

A method for tuning of PID and fractional order PID controller has been proposed. The presented method is based on idea of using Zeigler-Nichols for K_p and K_i while Astrom-Hagglund method is used for determining K_d for the conventional PID. Similarly K_p , K_d and K_i parameter for fractional order PID controller have been computed from Zeigler and Nichols method and new design specification and the remaining parameter λ and μ have been found by using Genetic Algorithm. From the unit step response different parameter are calculated. But to find out the optimal value of derivative order and integer order this paper takes the help of genetic algorithm. With the help of Genetic Algorithm based fractional order PID controller, control system responses can be designed with much more flexibility.

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