Modelling Liquor Transmission: Curtail through Optimal Control

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Abstract

In this paper, a mathematical model has been formulated as a system of non-linear ordinary differential equations to curtail the transmission of liquor habit among male and female individuals through optimal control. A basic reproduction number has been calculated at a liquor free equilibrium point and the rest three cases of equilibrium points that are when only male liquors, when only female liquors and when both liquors have also been evaluated. Stability analysis has been carried out for all the four cases of equilibrium points. Optimization of control has been carried out which were applied on infected male and female. Simulation has been carried out for all the compartments to analyze the impact of control numerically.

Keywords: *Mathematical Model, Transmission, Liquor, Equilibrium points, Control, Numerical Simulation*

1. Introduction

In today's world, researchers have studied that women have caught up with men in the field of drinking alcohol. They have also added that this is particularly true for women born in the last 15 to 25 years. The gap of drinking habits among male and female individuals is close to each other. The study was not designed to determine the closing gap is due to whom. After the year 2000, it is observed that the rate of consumption of liquor in male is just 1.1 times more than that of women. In decades past, male used to consume more than that of female [7].

When it comes to processing alcohol, male and female on consuming equal amount of alcohol although on having similar height and weight does not experience similar symptoms. It has been observed that women are adversely affected than men. Thus, it becomes necessary for every female individual to know this difference and reduce their consumption. Men have more blood volume, less body fat and also have higher concentration of dehydrogenase-an enzyme that breaks down alcohol than women. So, males are able to handle there body state on consumption than females. Thus, females respond faster. Numerous parts of body are affected due to this habit of liquor among both male and female individuals due to which they become victim of various diseases and liquor related ailments which exhibits varying levels. Researchers have said from their studies that women should take particular concern regarding it as it is prone for negative health effects [6], [9].

Robinson has added that drinking too much, too often can store up in both physical and mental future health problems, with people not realizing how easy it is to go over recommended limits. This is the reason why mandatory health warnings on alcohol products and a mass media campaign declared by medical officers should be widely known and understood [8].

Shah N.H. *et al* [13] has carried out their research on "Liquor Habit Transmission Model" in which the population dynamics of liquor habit based on number of pegs taken by an individual in a day was analyzed. Also Shah N.H.*et al* [12] has developed a model entitled "Mathematical Model for curbing liquor habit through Rehabilitation" in which they have described how an individual who have become victim of illness joins rehabilitation and quit themselves from this harmful habit.

In this paper, a mathematical model has been constructed to decrease the transmission of liquor habit through optimal control. Here, how the habit transfers from each other by coming in contact has also been discussed. Model formulation is done in Section 2. Stability has been discussed in Section 3. Section 4 consists of optimal control part which consists of how much and on whom the control is applied is calculated. Its numerical analysis has been carried out in Section 5. Conclusion has been included in Section 6.

2. Mathematical Model

Here, we formulate a mathematical model for causing of liquor related illness in male and female individuals. The notations along with its parametric values are given in Table 1.

Table 1: Notations and Parametric Values

Notations	Description	Parametric Values
S_M	Male susceptible to liquor	60
E_M	Male who consume less than 2 pegs per day	40
I_M	Male who consume more than 2 pegs per day	16
S_F	Female susceptible to liquor	40
E_F	Female who consume less than 2 pegs per day	30
I_F	Female who consume more than 2 pegs per day	12
L_I	Liquor related illness	10
B_M	New Recruitment rate of male	0.2
B_F	New Recruitment rate of female	0.1
β_M	Rate at which susceptible male are exposed	0.2
β_F	Rate at which susceptible female are exposed	0.1
ϕ_M	Rate at which exposed male start taking more than 2 pegs per day	0.3
δ_M	Rate at which the infected male becomes victim of liquor-related illness	0.4
δ_F	Rate at which the infected female becomes victim of liquor-related illness	0.2
σ	Rate at which exposed female start taking more than 2 pegs per day	0.1
γ_M	Rate at which the infected male comes in contact with susceptible female	0.1
γ_F	Rate at which the infected female comes in contact with susceptible male	0.03
μ	Natural Mortality Rate	0.5
$u_1(t)$	Control in terms of effort required to discourage infected male to interact with susceptible female	[0,1]
$u_2(t)$	Control in terms of effort required to discourage infected female to interact with susceptible male	[0,1]
$u_3(t)$	Control in terms of effort needed to aware the infected female individuals regarding health issues and encourage them to stop liquoring	[0,1]

The flow diagram of male and female becoming victim of illness is as shown in Figure 1.

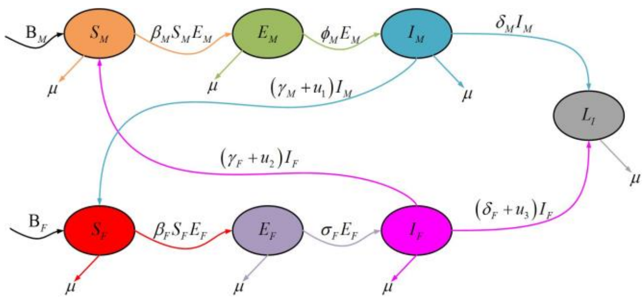

Figure 1: Flow Diagram of Liquor Habits in male and female and its related illness

In the beginning, male susceptible to liquor (S_M) gets little exposed of their habit at the rate (β_M) and starts taking less than 2 pegs per day (E_M). Then day by day this habit of liquoring increases at the rate (ϕ_M) and these male individuals becomes infected of taking liquor more than 2 pegs per day (I_M). Similarly, female susceptible to liquor (S_F) gets little exposed of their habit at the rate (β_F) and starts taking less than 2 pegs per day (E_F) initially. Then day by day this habit of liquoring increases at the rate (σ) and these female individuals becomes infected of taking liquor more than 2 pegs per day (I_F). At some stage of the life these individuals i.e. both male and female becomes victim of illness (L_I) at the rate δ_M and δ_F respectively. Rates γ_M and γ_F are described as interactives when infected male comes in contact with susceptible female and infected females comes in contact with susceptible males respectively. B_M and B_F describes the newly recruitment rate of males and females respectively and μ describes natural death rate of individuals and N denotes total human population.

Now, from the above figure 1 we have the following set of nonlinear ordinary differential equations describing the motion of individuals from one compartment to other.

$$\frac{dS_M}{dt} = \mathrm{B}_M - \beta_M S_M E_M + \gamma_F I_F - \mu S_M$$
$$\frac{dE_M}{dt} = \beta_M S_M E_M - \phi_M E_M - \mu E_M$$
$$\frac{dI_M}{dt} = \phi_M E_M - \delta_M I_M - \gamma_M I_M - \mu I_M$$
$$\frac{dS_F}{dt} = \mathrm{B}_F - \beta_F S_F E_F + \gamma_M I_M - \mu S_F$$
$$\frac{dE_F}{dt} = \beta_F S_F E_F - \sigma_F E_F - \mu E_F$$
$$\frac{dI_F}{dt} = \sigma_F E_F - \delta_F I_F - \gamma_F I_F - \mu I_F$$
$$\frac{dL_I}{dt} = \delta_M I_M + \delta_F I_F - \mu L_I \quad (1)$$

with $S_M + E_M + I_M + S_F + E_F + I_F + L_I \le N$ and $S_M, S_F > 0;\ E_M, I_M, E_F, I_F, L_I \ge 0$.

On adding the above set of equations (1) we get

$$\frac{d}{dt}(S_M + E_M + I_M + S_F + E_F + I_F + L_I)$$
$$= \mathrm{B}_M + \mathrm{B}_F - \mu(S_M + E_M + I_M + S_F + E_F + I_F + L_I) \ge 0$$

Thus, Liquor-Illness free equilibrium point of the model is $E_0 = \left(\frac{\mathrm{B}_M}{\mu}, 0, 0, \frac{\mathrm{B}_F}{\mu}, 0, 0, 0\right)$.

Now, we are interested in calculating the basic reproduction number which is to be calculated using next generation matrix method [1], [2], [4], [14]. The next generation matrix method is defined as FV^{-1} where F and V both are Jacobian matrices of $\Im$ and v evaluated with respect to liquor exposed and infected males and females and illness due to it in them at the point E_0.

Let $X = (E_M, I_M, S_F, E_F, I_F, L_I, S_M)$

$$\therefore \frac{dX}{dt} = \Im(X) - v(X)$$

where $\Im(X)$ denotes the rate of newly recruited and $v(X)$ denotes the rate of derived recruited which is given as

$$\Im(X) = \begin{bmatrix} \beta_M S_M E_M \\ 0 \\ 0 \\ \beta_F S_F E_F \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{and}$$

$$v(X) = \begin{bmatrix} \phi_M E_M + \mu I_M \\ -\phi_M E_M + (\delta_M + \gamma_M + \mu) I_M \\ -\mathrm{B}_F + \beta_F S_F E_F - \gamma_M I_M + \mu S_F \\ (\sigma + \mu) E_F \\ -\sigma E_F + (\delta_F + \gamma_F + \mu) I_F \\ -\delta_M I_M - \delta_F I_F + \mu L_I \\ -\mathrm{B}_M + \beta_M S_M E_M - \gamma_F I_F + \mu S_M \end{bmatrix}$$

Now, the derivative of $\Im$ and v evaluated at a Liquor-Illness free equilibrium point (E_0) gives matrices F and V of order 7×7 which is defined as

$$F = \left[\frac{\partial \Im_i(E_0)}{\partial X_j}\right] \quad V = \left[\frac{\partial v_i(E_0)}{\partial X_j}\right] \quad \text{for } i, j = 1,2,3,4,5,6,7$$

So,

$$F = \begin{bmatrix} \frac{\beta_M \mathrm{B}_M}{\mu} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\beta_M \mathrm{B}_M}{\mu} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{and}$$

$$V = \begin{bmatrix} \phi_M & \mu & 0 & 0 & 0 & 0 & 0 \\ -\phi_M & \delta_M + \gamma_M + \mu & 0 & 0 & 0 & 0 & 0 \\ 0 & -\gamma_M & \mu & \frac{\beta_F \mathrm{B}_F}{\mu} & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma + \mu & 0 & 0 & 0 \\ 0 & 0 & 0 & -\sigma & \delta_F + \gamma_F + \mu & 0 & 0 \\ 0 & -\delta_M & 0 & 0 & -\delta_F & \mu & 0 \\ \frac{\beta_M \mathrm{B}_M}{\mu} & 0 & 0 & 0 & -\gamma_F & 0 & \mu \end{bmatrix}$$

where V is non-singular matrix. Thus, the basic reproduction number R_0 which is the spectral radius of matrix FV^{-1} is given as

$$R_0=\frac{(\sigma+\mu)\left[\beta_M \mathrm{B}_M(\delta_M+\gamma_M+\mu)\right]+\phi_M\beta_F\mathrm{B}_F(\delta_M+\gamma_M+2\mu)}{\mu(\sigma+\mu)\phi_M(\delta_M+\gamma_M+2\mu)}$$

On equating the set of equations (1) to zero, we get four equilibrium points namely

1. Liquor-Illness free equilibrium point $(E_0)=(S_M,0,0,S_F,0,0,0)$
   where $S_M=\frac{\mathrm{B}_M}{\mu}$ and $S_F=\frac{\mathrm{B}_F}{\mu}$
2. When female do not liquor $(E_1)=(S_{M_1},E_{M_1},I_{M_1},S_{F_1},0,0,L_{I_1})$
   where $S_{M_1}=\frac{A_1}{\beta_M}$, $E_{M_1}=\frac{\beta_M\mathrm{B}_M-A_1\mu}{\beta_M A_1}$,
   $I_{M_1}=\frac{\phi_M E_M}{A_2}$,
   $S_{F_1}=\frac{\beta_M\phi_M\mathrm{B}_M\gamma_M+\beta_M A_1A_2\mathrm{B}_F-\phi_M A_1\gamma_M\mu}{\beta_M A_1A_2\mu}$ and
   $L_{I_1}=\frac{\alpha_1\phi_M E_M}{\mu A_2}$
3. When male do not liquor $(E_2)=(S_{M_2},0,0,S_{F_2},E_{F_2},I_{F_2},L_{I_2})$
   where, $S_{M_2}=\frac{A_3A_4\mathrm{B}_M\beta_F-A_3\gamma_F\mu\sigma+\mathrm{B}_F\gamma_F\sigma\beta_F}{A_3A_4\mu\beta_F}$,
   $S_{F_2}=\frac{A_3}{\beta_F}$, $E_{F_2}=-\frac{A_3\mu-\beta_F\mathrm{B}_F}{A_3\beta_F A_4}$, $I_{F_2}=\frac{\sigma E_F}{A_4}$ and
   $L_{I_2}=\frac{\delta_F\sigma E_F}{\mu A_4}$
4. When all exist which is known to be Endemic Equilibrium Point is defined as $(E^*)=(S_M{}^*,E_M{}^*,I_M{}^*,S_F{}^*,E_F{}^*,I_F{}^*,L_I{}^*)$
   where
   $S_M{}^*=\frac{A_1}{\beta_M}$,
   $$E_M{}^*=\frac{-A_2\begin{pmatrix}\beta_M A_3A_4\mathrm{B}_M\beta_F-\beta_M A_3\gamma_F\mu\sigma\\+\beta_M\mathrm{B}_F\gamma_F\sigma\beta_F-A_1A_3A_4\mu\beta_F\end{pmatrix}}{\beta_M\beta_F(\phi_M\gamma_M\gamma_F\sigma-A_1A_2A_3A_4)},$$
   $I_M{}^*=\frac{\phi_M E_M}{A_2}$
   $S_F{}^*=\frac{A_3}{\beta_F}$
   $$E_F{}^*=\frac{-A_4\begin{pmatrix}\beta_M\phi_M\mathrm{B}_M\gamma_M\beta_F-\beta_M A_1A_2A_3\mu\\+\beta_M A_1A_2\mathrm{B}_F\beta_F-\phi_M A_1\gamma_M\mu\beta_F\end{pmatrix}}{\beta_M\beta_F(\phi_M\gamma_M\gamma_F\sigma-A_1A_2A_3A_4)}$$
   $I_F{}^*=\frac{\sigma E_F}{A_4}$ and
   $L_I{}^*=\alpha_1 I_M+\alpha_2 I_F$.

In all, A_1,A_2,A_3,A_4 which are used above in four equilibrium points are

$$A_1=\phi_M+\mu,$$
$$A_2=\delta_M+\gamma_M+\mu,$$
$$A_3=\sigma+\mu,$$
$$A_4=\delta_F+\gamma_F+\mu \qquad (2)$$

3. Stability Analysis

In this section, the local and global stability at E_0, E_1, E_2 and E^* are to be studied using the linearization method and matrix analysis.

3.1 Local Stability

Theorem 3.1.1: (stability at E_0) If $\frac{\beta_M\mathrm{B}_M-A_1\mu}{\mu}<0$ then a system is locally asymptotically stable at liquor-illness free equilibrium point.

Proof: At point E_0, the Jacobian matrix of the system (1) is

$$J(E_0)=\begin{bmatrix}-\mu & -\frac{\beta_M\mathrm{B}_M}{\mu} & 0 & 0 & 0 & \gamma_F & 0\\ 0 & \frac{\beta_M\mathrm{B}_M}{\mu}-A_1 & 0 & 0 & 0 & 0 & 0\\ 0 & \phi_M & -A_2 & 0 & 0 & 0 & 0\\ 0 & 0 & \gamma_M & -\mu & -\frac{\beta_F\mathrm{B}_F}{\mu} & 0 & 0\\ 0 & 0 & 0 & 0 & \frac{\beta_F\mathrm{B}_F}{\mu}-A_3 & 0 & 0\\ 0 & 0 & 0 & 0 & \sigma & -A_4 & 0\\ 0 & 0 & \delta_M & 0 & 0 & \delta_F & -\mu\end{bmatrix}$$

where A_1,A_2,A_3,A_4 are defined same as equation (2)

The eigenvalues of the above matrix are

$\lambda_1=-A_4<0$,

$\lambda_2 = -\dfrac{A_3\mu - B_F\beta_F}{\mu} < 0$,

$\lambda_3 = -A_2 < 0$, $\lambda_4 = -\mu < 0$, $\lambda_5 = -\mu < 0$, $\lambda_6 = -\mu < 0$ and

$\lambda_7 = \dfrac{\beta_M B_M - A_1\mu}{\mu}$.

So, by Routh Hurwitz criteria [5], if $\dfrac{\beta_M B_M - A_1\mu}{\mu} < 0$ then a system is locally asymptotically stable at this point E_0 .

Theorem 3.1.2: (stability at E_1) The system is locally asymptotically stable at an equilibrium point when female do not liquor .

Proof: At point E_1, the Jacobian matrix of the system (1) is

$$J(E_1) = \begin{bmatrix} -A_5-\mu & A_6 & 0 & 0 & 0 & \gamma_F & 0 \\ A_5 & -A_6-A_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \phi_M & -A_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \gamma_M & -\mu & A_8 & 0 & 0 \\ 0 & 0 & 0 & 0 & -A_8-A_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma & -A_4 & 0 \\ 0 & 0 & \delta_M & 0 & 0 & \delta_F & -\mu \end{bmatrix}$$

where A_1, A_2, A_3, A_4 are defined same as equation (2) and

$$A_5 = \beta_M E_M^{\ *},$$
$$A_6 = -\beta_M S_M^{\ *},$$
$$A_7 = \beta_M E_F^{\ *},$$
$$A_8 = -\beta_F S_F^{\ *} \qquad (3)$$

The eigenvalues of the above matrix are $\lambda_1, \lambda_2, \lambda_3, \lambda_4 = -\mu < 0$ and the rest λ_5, λ_6 and λ_7 satisfies the characteristic equation $a_0\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0$

where $a_0 = 1 > 0$,

$a_1 = A_8 + A_3 + A_6 + A_1 + A_5 + \mu > 0$,

$$a_2 = A_1A_3 + A_1A_5 + A_1A_8 + A_1\mu + A_3A_5 + A_3A_6 + A_3\mu + A_5A_8 + A_6A_8 + A_6\mu + A_8\mu > 0$$

$$a_3 = (A_8 + A_3)(A_1A_5 + A_1\mu + A_6\mu) > 0$$

As a_0, a_1, a_2 and a_3 are all greater than 0 and also $a_1a_2 > a_3$

Hence, all the conditions of Routh Hurwitz criteria are satisfied and this shows that λ_5, λ_6 and λ_7 have negative real part.

Thus, by Routh Hurwitz criteria the system proves to be locally asymptotically stable at E_1.

Theorem 3.1.3: (stability at E_2) The system is locally asymptotically stable at an equilibrium point when male do not liquor .

Proof: At point E_2, the Jacobian matrix of the system (1) is

$$J(E_2) = \begin{bmatrix} -\mu & A_6 & 0 & 0 & 0 & \gamma_F & 0 \\ 0 & -A_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \phi_M & -A_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \gamma_M & -A_7-\mu & A_8 & 0 & 0 \\ 0 & 0 & 0 & A_7 & -A_8-A_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma & -A_4 & 0 \\ 0 & 0 & \delta_M & 0 & 0 & \delta_F & -\mu \end{bmatrix}$$

where $A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8$ are defined as in equation (2) and (3) above.

The eigen values of the above matrix are $\lambda_1 = -A_4 < 0$, $\lambda_2 = -A_2 < 0$, $\lambda_3 = -A_1 < 0$, $\lambda_4, \lambda_5 = -\mu < 0$ and the rest λ_6 , λ_7 satisfies the characteristic equation $a_0\lambda^2 + a_1\lambda + a_2 = 0$

where $a_0 = 1 > 0$,

$a_1 = A_3 + A_7 + A_8 + \mu > 0$,

$a_2 = A_3A_7 + \mu A_3 + \mu A_8 > 0$

Hence, by Routh Hurwitz criteria the system is locally asymptotically stable at point E_2.

Theorem 3.1.4: (stability at E^*) The system is locally asymptotically stable at an endemic equilibrium point where everyone exists. .

Proof: At point E^*, the Jacobian matrix of the system (1) is

$$J(E_3) = \begin{bmatrix} -A_5-\mu & A_6 & 0 & 0 & 0 & \gamma_F & 0 \\ A_5 & A_6-A_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \phi_M & -A_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \gamma_M & -A_7-\mu & A_8 & 0 & 0 \\ 0 & 0 & 0 & A_7 & -A_8-A_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma & -A_4 & 0 \\ 0 & 0 & \delta_M & 0 & 0 & \delta_F & -\mu \end{bmatrix}$$

where $A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8$ are defined as in equation (2) and (3) above.

The eigenvalue of the above matrix is $\lambda_1 = -\mu < 0$ and the rest $\lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7$ satisfies the characteristic equation $a_0\lambda^6 + a_1\lambda^5 + a_2\lambda^4 + a_3\lambda^3 + a_4\lambda^2 + a_5\lambda + a_6 = 0$

where $a_0 = 1 > 0$

$$a_1 = A_4 + A_8 + A_3 + A_7 + A_2 + A_6 + A_1 + A_5 + 2\mu > 0 \quad ,$$

$$\begin{aligned} a_2 = {} & A_1\left(A_2 + A_3 + A_4 + A_5 + A_7 + A_8 + 2\mu\right) \\ & + A_2\left(A_3 + A_4 + A_5 + A_6 + A_7 + A_8 + 2\mu\right) \\ & + A_3\left(A_4 + A_5 + A_6 + A_7 + 2\mu\right) \\ & + A_4\left(A_5 + A_6 + A_7 + A_8 + 2\mu\right) \\ & + A_5\left(A_7 + A_8 + \mu\right) \\ & + A_6\left(A_7 + A_8 + 2\mu\right) + A_7\mu + 2A_8\mu + \mu^2 > 0 \end{aligned}$$

$$\begin{aligned} a_3 = {} & A_1\left\{\begin{matrix} A_2\left(A_3 + A_4 + A_5 + A_7 + A_8 + 2\mu\right) + A_3\left(A_4 + A_5 + A_7 + 2\mu\right) \\ + A_4\left(A_5 + A_7 + A_8 + 2\mu\right) + A_5\left(A_7 + A_8 + \mu\right) + \mu\left(A_7 + 2A_8 + \mu\right) \end{matrix}\right\} \\ & + A_2\left\{\begin{matrix} A_3\left(A_4 + A_5 + A_6 + A_7 + 2\mu\right) + A_4\left(A_5 + A_6 + A_7 + A_8 + 2\mu\right) \\ + A_5\left(A_7 + A_8 + \mu\right) + A_6\left(A_7 + A_8 + 2\mu\right) + \mu\left(A_7 + 2A_8 + \mu\right) \end{matrix}\right\} \\ & + A_3\left\{\begin{matrix} A_4\left(A_5 + A_6 + A_7 + 2\mu\right) + A_5\left(A_7 + \mu\right) \\ + A_6\left(A_7 + 2\mu\right) + \mu\left(A_7 + \mu\right) \end{matrix}\right\} \\ & + A_4\left\{\begin{matrix} A_5\left(A_7 + A_8 + \mu\right) + A_6\left(A_7 + A_8 + 2\mu\right) \\ + \mu\left(A_7 + 2A_8 + \mu\right) \end{matrix}\right\} \\ & + A_6\left\{\mu\left(A_7 + 2A_8 + \mu\right)\right\} + A_8\mu\left(A_5 + \mu\right) > 0 \end{aligned}$$

$$\begin{aligned} a_4 = {} & A_1\left[\begin{matrix} A_2\left\{\begin{matrix} A_3\left(A_4 + A_5 + A_7 + 2\mu\right) + A_4\left(A_5 + A_7 + A_8 + 2\mu\right) \\ + A_5\left(A_7 + A_8 + \mu\right) + \mu\left(A_7 + 2A_8 + \mu\right) \end{matrix}\right\} \\ + A_3\left\{A_4\left(A_5 + A_7 + 2\mu\right) + A_5\left(A_7 + \mu\right) + \mu\left(A_7 + \mu\right)\right\} \\ + \mu\left\{A_4\left(A_7 + 2A_8 + \mu\right) + A_8\left(A_5 + \mu\right)\right\} \end{matrix}\right] \\ & + A_2\left[\begin{matrix} A_3\left\{\begin{matrix} A_4\left(A_5 + A_6 + A_7 + 2\mu\right) + A_5\left(A_7 + \mu\right) \\ + A_6\left(A_7 + 2\mu\right) + \mu\left(A_7 + \mu\right) \end{matrix}\right\} \\ + A_4\left\{\begin{matrix} A_5\left(A_7 + A_8 + \mu\right) + A_6\left(A_7 + A_8 + 2\mu\right) \\ + \mu\left(A_7 + 2A_8 + \mu\right) \end{matrix}\right\} \\ + \mu\left\{A_8\left(A_5 + \mu\right) + A_6\left(A_7 + 2A_8 + \mu\right)\right\} \end{matrix}\right] \\ & + A_3\left[A_4\left\{A_5\left(A_7 + \mu\right) + A_6\left(A_7 + 2\mu\right) + \mu\left(A_7 + \mu\right)\right\}\right] \\ & + \mu\left[A_6\left\{A_3\left(A_7 + \mu\right) + A_4\left(A_5A_8 + A_6\left(A_7 + 2A_8 + \mu\right) + A_8\mu\right)\right\}\right] \\ & + A_6A_8\mu^2 > 0 \end{aligned}$$

$$\begin{aligned} a_5 = {} & A_1A_2A_3\left[A_4\left(A_5 + A_7 + 2\mu\right) + A_5\left(A_7 + \mu\right) + \mu\left(A_7 + \mu\right)\right] \\ & + A_1A_2\left[\begin{matrix} A_4\left\{A_5\left(A_7 + A_8 + \mu\right) + \mu\left(A_7 + 2A_8 + \mu\right)\right\} \\ + A_8\mu\left(A_5 + \mu\right) \end{matrix}\right] \\ & + A_1A_4\left[A_3\left\{A_5\left(A_7 + \mu\right) + \mu\left(A_7 + \mu\right)\right\} + A_8\mu\left(A_5 + \mu\right)\right] \\ & + A_2A_4\left[\begin{matrix} A_3\left\{A_5\left(A_7 + \mu\right) + A_6\left(A_7 + 2\mu\right)\right\} \\ + \mu\left\{A_8\left(A_5 + 2A_6 + \mu\right) + A_6\left(A_7 + \mu\right)\right\} \end{matrix}\right] \\ & + A_6\mu\left[A_8\mu\left(A_2 + A_4\right) + A_3A_4\left(A_7 + \mu\right)\right] > 0 \end{aligned}$$

$$\begin{aligned} a_6 = {} & A_1A_2\left[A_3A_4\left\{A_5\left(A_7 + \mu\right) + \mu\left(A_7 + \mu\right)\right\} + A_4A_8\mu\left(A_5 + \mu\right)\right] \\ & + A_2A_4A_6\left[A_3\mu\left(A_7 + \mu\right) + A_8\mu^2\right] - \sigma\gamma_M\phi_M A_5\gamma_F A_7 \\ & > 0 \end{aligned}$$

By, Routh Hurwitz criteria this positive $a_1, a_2, a_3, a_4, a_5, a_6$ shows that $\lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7$ have negative real part and all the conditions of Routh Hurwitz criteria are satisfied for n=6.

Thus, the system is locally asymptotically stable at E^*.

3.2 Global Stability

Theorem 3.2.1: (stability at E_0) If $\min\left\{\beta_M \mathrm{B}_M, \beta_F \mathrm{B}_F\right\} \le \mu^2$ then the system is globally asymptotically stable at liquor-illness free equilibrium point.

Proof: Consider a Lyapunov function

$$L(t) = E_M(t) + I_M(t) + E_F(t) + I_F(t) + L_I(t)$$

$$\therefore L'(t) = \left(\beta_M S_M - \mu\right)E_M + \left(\beta_F S_F - \mu\right)E_F - \mu\left[I_M + I_F + L_I\right] - \gamma_M I_M - \gamma_F I$$

Since $E_0 \in \Lambda$, thus $S_M \le \dfrac{\mathrm{B}_M}{\mu}$ and $S_F \le \dfrac{\mathrm{B}_F}{\mu}$ so we have

$$\begin{aligned} L'(t) \le {} & \left(\frac{\beta_M \mathrm{B}_M}{\mu} - \mu\right)E_M + \left(\frac{\beta_F \mathrm{B}_F}{\mu} - \mu\right)E_F \\ & - \mu\left[I_M + I_F + L_I\right] - \gamma_M I_M - \gamma_F I_F \le 0 \end{aligned}$$

if $\min\left\{\beta_M \mathrm{B}_M, \beta_F \mathrm{B}_F\right\} \le \mu^2$.

Moreover, as μ is non-negative it follows that $L'(t) < 0$ if $\min\left\{\beta_M \mathrm{B}_M, \beta_F \mathrm{B}_F\right\} \le \mu^2$ and

$$L'(t) = 0 \text{ if } E_M = I_M = E_F = I_F = L_I = 0.$$

Hence, the only solution of system (1) in which $L'(t) = 0$ is E_0 in Λ.

Therefore by Lasalle's Invariance Principle [10], every solution of system (1) with initial conditions in Λ, approaches E_0 as $t \to \infty$.

Hence, the system is globally asymptotically stable at E_0.

Theorem 3.2.2: (stability at E_1) If $\beta_M \mathrm{B}_M \le \mu^2$ then the system is globally asymptotically stable at an equilibrium point when female do not liquor.

Proof: Consider a Lyapunov function

$$L(t) = E_M(t) + I_M(t) + L_I(t)$$

$$\therefore L'(t) = \left(\beta_M S_M - \mu\right)E_M - \left[\left(\gamma_M + \mu\right)I_M + \mu L_I\right]$$

Since $E_0 \in \Lambda$, thus $S_M \le \dfrac{\mathrm{B}_M}{\mu}$ and we have

$$L'(t) \le \left(\frac{\beta_M \mathrm{B}_M}{\mu} - \mu\right)E_M - \left[(\gamma_M + \mu)I_M + \mu L_I\right] \le 0$$ if $\beta_M \mathrm{B}_M \le \mu^2$.

Moreover, as μ is non-negative it follows that $L'(t) < 0$ if $\beta_M \mathrm{B}_M \le \mu^2$ and

$L'(t) = 0$ if $E_M = I_M = L_I = 0$.

Hence, the only solution of system (1) in which $L'(t) = 0$ is E_0 in Λ.

Therefore by Lasalle's Invariance Principle, every solution of system (1) with initial conditions in Λ, approaches E_0 as $t \to \infty$.

Hence, the system is globally asymptotically stable at E_1.

Theorem 3.2.3: (stability at E_2) If $\beta_F \mathrm{B}_F \le \mu^2$ then the system is globally asymptotically stable at an equilibrium point when male do not liquor.

Proof: Consider a Lyapunov function

$$L(t) = E_F(t) + I_F(t) + L_I(t)$$

$$\therefore L'(t) = (\beta_F S_F - \mu)E_F - \left[(\gamma_F + \mu)I_F + \mu L_I\right]$$

Since $E_0 \in \Lambda$, thus $S_F \le \frac{\mathrm{B}_F}{\mu}$ and we have

$$L'(t) \le \left(\frac{\beta_F \mathrm{B}_F}{\mu} - \mu\right)E_F - \left[(\gamma_F + \mu)I_F + \mu L_I\right] \le 0$$

if $\beta_F \mathrm{B}_F \le \mu^2$.

Moreover, as μ is non-negative it follows that $L'(t) < 0$ if $\beta_F \mathrm{B}_F \le \mu^2$ and

$L'(t) = 0$ if $E_F = I_F = L_I = 0$.

Hence, the only solution of system (1) in which $L'(t) = 0$ is E_0 in Λ.

Therefore by Lasalle's Invariance Principle, every solution of system (1) with initial conditions in Λ, approaches E_0 as $t \to \infty$.

Hence, the system is globally asymptotically stable at E_2.

Theorem 3.2.4: (stability at E^*) The system is globally asymptotically stable at an endemic equilibrium point where everyone exists.

Proof: Consider a Lyapunov function

$$L(t) = \frac{1}{2}\begin{bmatrix} \{S_M(t) - S_M^*(t)\} + \{E_M(t) - E_M^*(t)\} \\ + \{I_M(t) - I_M^*(t)\} + \{S_F(t) - S_F^*(t)\} \\ + \{E_F(t) - E_F^*(t)\} + \{I_F(t) - I_F^*(t)\} \\ + \{L_I(t) - L_I^*(t)\} \end{bmatrix}^2$$

$$\therefore L'(t) = \begin{bmatrix} \{S_M - S_M^*\} + \{E_M - E_M^*\} + \{I_M - I_M^*\} \\ + \{S_F - S_F^*\} + \{E_F - E_F^*\} + \{I_F - I_F^*\} + \{L_I - L_I^*\} \end{bmatrix} \left[S_M' + E_M' + I_M' + S_F' + E_F' + L_I'\right]$$

$$= \begin{bmatrix} \{S_M - S_M^*\} + \{E_M - E_M^*\} + \{I_M - I_M^*\} \\ + \{S_F - S_F^*\} + \{E_F - E_F^*\} + \{I_F - I_F^*\} + \{L_I - L_I^*\} \end{bmatrix} \begin{bmatrix} \mu\{S_M^* + E_M^* + I_M^* + S_F^* + E_F^* + L_I^*\} \\ -\mu\{S_M + E_M + I_M + S_F + E_F + L_I\} \end{bmatrix}$$

$$= -\mu \begin{bmatrix} \{S_M - S_M^*\} + \{E_M - E_M^*\} + \{I_M - I_M^*\} \\ + \{S_F - S_F^*\} + \{E_F - E_F^*\} + \{I_F - I_F^*\} + \{L_I - L_I^*\} \end{bmatrix}^2 < 0$$

where, we have used

$\mathrm{B}_M + \mathrm{B}_F = \mu\left(S_M^* + E_M^* + I_M^* + S_F^* + E_F^* + I_F^* + L_I^*\right)$.

Hence, the system is globally asymptotically stable at E^*.

4. Optimal Control Model

In this section a control function has been implemented on infected male and female individuals to decrease liquor related illness in the society. The system of non-linear ordinary differential equations for this is as follows:

$$\frac{dS_M}{dt} = \mathrm{B}_M - \beta_M S_M E_M + (\gamma_F + u_2)I_F - \mu S_M$$

$$\frac{dE_M}{dt} = \beta_M S_M E_M - \phi_M E_M - \mu E_M$$

$$\frac{dI_M}{dt} = \phi_M E_M - \delta_M I_M - (\gamma_M + u_1)I_M - \mu I_M$$

$$\frac{dS_F}{dt} = \mathrm{B}_F - \beta_F S_F E_F + (\gamma_M + u_1)I_M - \mu S_F$$

$$\frac{dE_F}{dt} = \beta_F S_F E_F - \sigma_F E_F - \mu E_F$$

$$\frac{dI_F}{dt} = \sigma_F E_F - \delta_F I_F - (\gamma_F + u_2)I_F - \mu I_F$$

$$\frac{dL_I}{dt} = \delta_M I_M + \delta_F I_F - \mu L_I$$

The objective function along with the optimal control variable for above system of equations is given by

$$J\left(u_i,\Omega\right)=\int_0^T\begin{pmatrix} A_1S_M{}^2+A_2E_M{}^2+A_3I_M{}^2 \\ +A_4S_F{}^2+A_5E_F{}^2+A_6I_F{}^2 \\ +A_7L_I{}^2+w_1u_1{}^2+w_2u_2{}^2+w_3u_3{}^2 \end{pmatrix}dt \qquad (4)$$

where Ω denotes the set of all compartmental variables, $A_1, A_2, A_3, A_4, A_5, A_6, A_7$ denotes non-negative weight constants for the compartments $S_M, E_M, I_M, S_F, E_F, I_F, L_I$ respectively and w_1, w_2, w_3 are the weight constant for the control variable u_1, u_2, u_3 respectively.

The weights w_1, w_2 and w_3 which are constant parameters for u_1, u_2 and u_3 will standardize the optimal control condition.

Now, we will calculate the value of control variables u_1, u_2 and u_3 from $t=0$ to $t=T$ such that

$$J\left(u_1(t),u_2(t),u_3(t)\right)=\min\left\{J\left(u_i^*,\Omega\right)/u_1,u_2,u_3\in\phi\right\}$$

where ϕ = smooth function on the interval $[0,1]$.

Using, Fleming and Rishel results [3], the optimal control denoted by u_i^* is obtained by collecting all the integrands of the objective function (4) using the lower bounds and upper bounds of the both the control variables respectively. Using Pontrygin's principle [11], we construct a Lagrangian function consisting of state equation and adjoint variables $A_V=(\lambda_{S_M},\lambda_{E_M},\lambda_{I_M},\lambda_{S_F},\lambda_{E_F},\lambda_{I_F},\lambda_{L_I})$ which is as follows:

$$\begin{aligned} L\left(\Omega,A_V\right) &= A_1S_M{}^2+A_2E_M{}^2+A_3I_M{}^2+A_4S_F{}^2 \\ &+A_5E_F{}^2+A_6I_F{}^2+A_7L_I{}^2+w_1u_1{}^2+w_2u_2{}^2+w_3u_3{}^2 \\ &+\lambda_{S_M}\left[\mathrm{B}_M-\beta_MS_ME_M+(\gamma_F+u_2)I_F-\mu S_M\right] \\ &+\lambda_{E_M}\left[\beta_MS_ME_M-\phi_ME_M-\mu E_M\right] \\ &+\lambda_{I_M}\left[\phi_ME_M-\delta_MI_M-(\gamma_M+u_1)I_M-\mu I_M\right] \\ &+\lambda_{S_F}\left[\mathrm{B}_F-\beta_FS_FE_F+(\gamma_M+u_1)I_M-\mu S_F\right] \\ &+\lambda_{E_F}\left[\beta_FS_FE_F-\sigma_FE_F-\mu E_F\right] \\ &+\lambda_{I_F}\left[\sigma_FE_F-\delta_FI_F-(\gamma_F+u_2)I_F-\mu I_F\right] \\ &+\lambda_{L_I}\left[\delta_MI_M+\delta_FI_F-\mu L_I\right] \end{aligned}$$

Now, the partial derivative of the Lagrangian function with respect to each variable of the compartment gives us the adjoint equation such that

$$\begin{aligned} \dot{\lambda}_{S_M} &= -\frac{\partial L}{\partial S_M} \\ &= -2A_1S_M+\beta_ME_M\left(\lambda_{S_M}-\lambda_{E_M}\right)+\mu\lambda_{S_M} \\ \dot{\lambda}_{E_M} &= -\frac{\partial L}{\partial E_M} \\ &= -2A_2E_M+\beta_MS_M\left(\lambda_{S_M}-\lambda_{E_M}\right)+\phi_M\left(\lambda_{E_M}-\lambda_{I_M}\right)+\mu\lambda_{E_M} \\ \dot{\lambda}_{I_M} &= -\frac{\partial L}{\partial I_M} \\ &= -2A_3I_M+\delta_M\left(\lambda_{I_M}-\lambda_{L_I}\right)+\left(\gamma_M+u_1\right)\left(\lambda_{I_M}-\lambda_{S_F}\right)+\mu\lambda_{I_M} \\ \dot{\lambda}_{S_F} &= -\frac{\partial L}{\partial S_F} \\ &= -2A_4S_F+\beta_FE_F\left(\lambda_{S_F}-\lambda_{E_F}\right)+\mu\lambda_{S_F} \\ \dot{\lambda}_{E_F} &= -\frac{\partial L}{\partial E_F} \\ &= -2A_5E_F+\beta_FS_F\left(\lambda_{S_F}-\lambda_{E_F}\right)+\sigma\left(\lambda_{E_F}-\lambda_{I_F}\right)+\mu\lambda_{E_F} \\ \dot{\lambda}_{I_F} &= -\frac{\partial L}{\partial I_F} \\ &= -2A_6I_F+\left(\delta_F+u_3\right)\left(\lambda_{I_F}-\lambda_{L_I}\right)+\left(\gamma_F+u_2\right)\left(\lambda_{I_F}-\lambda_{S_M}\right)+\mu\lambda_{I_F} \\ \dot{\lambda}_{L_I} &= -\frac{\partial L}{\partial L_I} \\ &= -2A_7L_I+\mu\lambda_{L_I} \end{aligned}$$

The necessary conditions for Lagrangian function L to be optimal for control are

$$\frac{\partial L}{\partial u_1}=2w_1u_1-I_M\left(\lambda_{I_M}-\lambda_{S_F}\right)=0 \qquad (5)$$

$$\frac{\partial L}{\partial u_2}=2w_2u_2-I_F\left(\lambda_{I_F}-\lambda_{S_M}\right)=0 \qquad (6)$$

$$\frac{\partial L}{\partial u_3}=2w_3u_3-I_F\left(\lambda_{I_F}-\lambda_{L_I}\right)=0 \qquad (7)$$

On solving equation (5), (6) and (7) we get,

$$u_1^*=\max\left(a_1,\min\left(b_1,\frac{I_M\left(\lambda_{I_M}-\lambda_{S_F}\right)}{2w_1}\right)\right)$$

$$u_2^*=\max\left(a_2,\min\left(b_2,\frac{I_F\left(\lambda_{I_F}-\lambda_{S_M}\right)}{2w_2}\right)\right)$$

$$u_3^*=\max\left(a_3,\min\left(b_3,\frac{I_F\left(\lambda_{I_F}-\lambda_{L_I}\right)}{2w_3}\right)\right)$$

where a_1, a_2, a_3 = lower bounds and b_1, b_2, b_3 = upper bounds of the control variables u_1, u_2 and u_3 respectively.

5. Numerical Simulation

In this section, we will analyze and study the effect of control on each compartment numerically using the data given in Table 1.

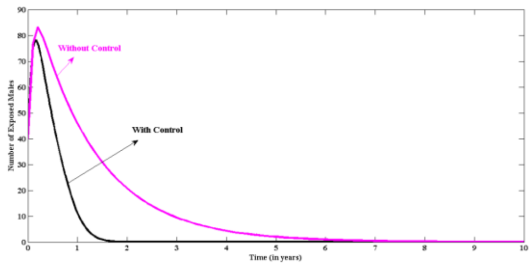

Figure 2: Effect of control on E_M compartment

Figure 2 depicts that when control is not applied number of exposed male increases from 40 to 85 approximately whereas these number of individuals decreases to about 79 approximately if control is applied and also in less duration as compared to without control.

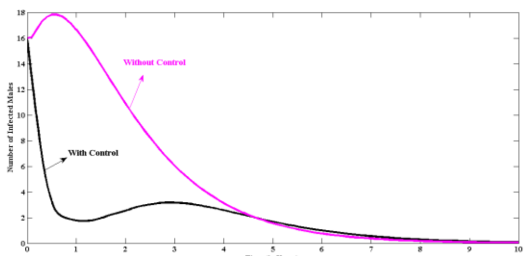

Figure 3: Effect of control on I_M compartment

Figure 3 shows that when control is applied these infected male individuals start to decrease their habit of liquoring within a year but then after they again starts which makes them victim of disease and thus they need to stop it due to their weak physical condition and so they again starts to decrease their habit after some time. In case of without control, they increases initially and after suffering they are forced by the body to decide on their own to stop this habit.

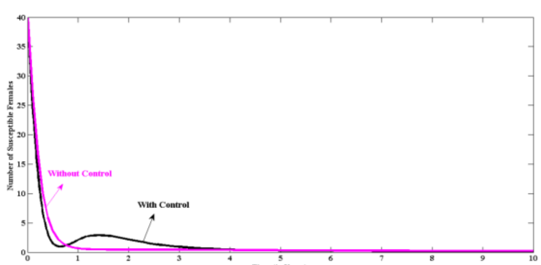

Figure 4: Effect of control on S_F compartment

Figure 4 interprets that initially in both the cases of control they are decreasing but control applied on females takes less time in comparison to without control. It is observed from the figure that the control applied on female increases for some time which is because they might have come in contact with the infected male which have forced them for liquoring, but then after the effect of control is seen as they again starts to decrease.

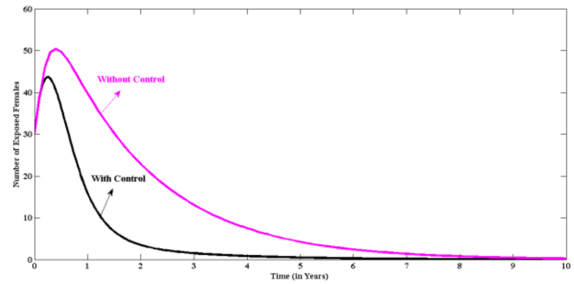

Figure 5: Effect of control on E_F compartment

It is seen from the figure 5 that in comparison to without control the exposed females soon tries to recover from their habit on control.

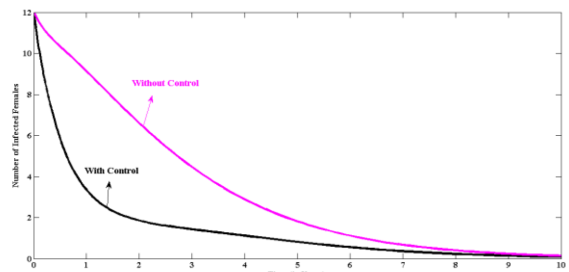

Figure 6: Effect of control on I_F compartment

Figure 6 depicts that infected females starts to adopt the path of decreasing their habit initially in comparison to without control as they know that it may not only cause liquor related illness but also many complications prevails in the body during their maturity stage. Also, here the control has been applied on infected females to stop convincing the male regarding the habit, so that these habit of liquoring among male and female decreases in the society.

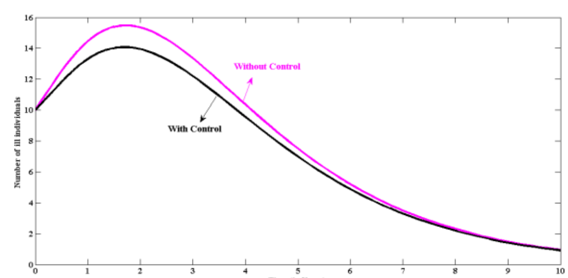

Figure 7: Effect of control on L_I compartment

Figure 7 shows the impact of control on L_I compartment. Many individuals in the society suffers from liquor related illness and these illness becomes difficult to cure. So, if control is applied, little relaxation from the disease can be observed among individuals from the figure in comparison to without control.

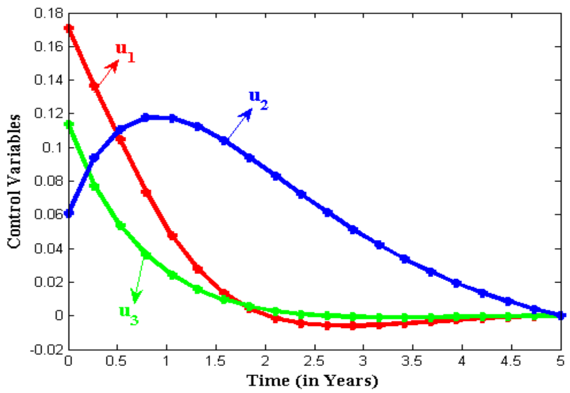

Figure 8: Control variables verses Time (in years)

Figure 8 shows the minimum requirement of all the three controls; u_1 , u_2 and u_3 with respect to time in years. It is observed that u_1 requires 17%, u_3 requires approximately 11.8% and u_2 requires 6 to 12% in individuals to become effective among male and female individuals.

6. Conclusion

Here, a paper has been formulated to understand the behavior of individuals in the society using mathematical model. Global Study has found that women have started drinking as much alcohol as men. But the complications in their illness are more in females as compared to man. The value of threshold which is calculated at liquor free equilibrium point is obtained as 0.21<1 which shows that 21% of the individuals get cured of their liquored habit through optimal control. If the value of threshold is greater than 1 then this don't happen. Also, the system has proved to be locally and globally asymptotically stable at all the four equilibrium points i.e. liquor free equilibrium point, when only male liquor, when only female liquors and liquor existence among male and female equilibrium point. Control applied on infected males and infected females has played a vital role in decreasing the habit among male and female individuals and also the illness due to it can also be decreased. The impact of control on each compartment can also be observed from the numerical simulation section.

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