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Structural Analysis and Identification of Isomorphism between Six Link Planar Kinematic Chains using Graph Theory

Shubham Verma[1], Vijay pal Singh[2]

[1,2] Mechanical Engineering, National Institute of Technology, Kurukshetra, Haryana 136119, India

Abstract

The process of mechanical design involves intuition, experience, analysis and synthesis. In recent time efforts have been made to develop an increasingly rational approach to engineering design. One of these involves the introduction of concept of graph theory in the structural analysis of mechanical systems. It is used for determining the structural characteristic of the kinematic chains such as isomorphism, and type of degree of freedom, viz., total, partial and fractionated. In this paper, an attempt has been made to provide satisfactory solutions to the structural aspects of a kinematic chain by considering the example of sewing machine mechanism. Structure of the kinematic chain defined by its kinematic graph and based on understanding of its function or working. The kinematic graphs defined by the vertex edge matrix and links are represented by vertex and joints by edges. This matrix is called adjacency matrix and represents the connection between the links. The proposed method is applied for determining the characteristic polynomial equation of this mechanism and compares it with straight line motion mechanism and brake drum mechanism for detecting isomorphism. Algebraic test is also done which is based on graph theory for determining total, partial and fractionated degree of freedom for the mechanism.

Keywords: *Kinematic graph, Isomorphism, Degree of Freedom, Binary/Ternary Links.*

1. Introduction

Graph theory plays very important role in the structural analysis and identification of isomorphism between the Kinematic chains and mechanisms. In this sense, mechanism and mechanical systems may be regarded as circuits with topological and net-work properties just as electrical and other system. Because of inherent simplicity, graph theory has a wide range of applications. In engineering, physical, social and biological science, linguistics and in numerous other areas, a graph can be used to represent almost any physical situation involving discrete objects and a relationship among them. There is a lot of work being done in this direction and also a plenty of work has already been done. A. H. Soni [1] performed structural analysis of two general constraint kinematic chains and their practical application. According to this paper, linkages with mobility one and mobility two with upto three loops were considered. A. H. Soni et al. [2] conducted application for linear and non linear graphs in structural synthesis of kinematic chains. They used the graph theory and polya's theory of counting and performed structural synthesis and analysis of planar and three dimensional kinematic chain. J. J. Uicker et al. [3] described a method for the identification and recognition of equivalence of kinematic chains and used the equality of identifying numbers which detected the kinematically equivalence of chains. H.S. Yan et al. [4] presented techniques by which the characteristic polynomial of kinematic chain and its coefficients were determined. T.S. Mruthyunjaya et al. [5] used the Bocher's formula for determining the characteristic polynomial for closed and connected kinematic graphs. V. P. Agarwal et al. [6] described two computationally simple and efficient analytical test for checking fractionated degree of freedom of the kinematic chains (FFKC) based on path loop connectivity matrix. V. P. Agarwal et al. [9] developed computationally simple and efficient analytical methods using matrices, link-link variable characteristic polynomial, link-link variable permanent function for the identification of isomorphism of kinematic chains and their mechanism such as path generators and function generators. J.N. Yadav et al. [10] used modified distance method for determining the isomorphism between among kinematic chains and mechanism. D. Huafeng et al. [13][15] presented a unique representation of graphs and on the basis of degree-sequences of loops, the perimeter graph was proposed which reduced the forms of graphs and adjacency matrices from hundreds of thousands to several or even just one.

The proposed method is simple and well suited for hand computation. An additional advantages of this method is that the Bocher's formula, when interpreted on the basis of well known results of graph theory, reveals the physical meaning of the characteristic co-efficients and thus lead to a possible way of arriving at these co-efficients by inspection of the chain itself. It uses the characteristic polynomial equations for structural analysis and the identification of isomorphism between the kinematic chains and mechanisms. This method is successfully applied to six link single degree of freedom planar kinematic chains.

According to graph theory two kinematic graphs are isomorphic if their vertex-edge matrix are equivalent or if all the links constituting the chains are identical. In the proposed method, characteristic polynomial equations are used for equality of the links if the characteristic equations of two kinematic graphs are same such that the graphs are isomorphic in nature. If the equations are not same then the graphs are not isomorphic.

2. Methodology Adopted

A kinematic chain is the assembly of links/pairs combination to form one or more closed loops. In this method graph theory is used to draw the configuration and tree diagrams of the given kinematic chain. With the help of the tree diagram the adjacency matrix is obtained and is known as vertex-edge matrix. In this matrix links are represented by vertices and joints by edges. The adjacency matrix is used in finding the characteristic polynomial of the given kinematic chain using Bocher's formula.

3. Structural Analysis of Sewing Machine Chain

The sewing machine mechanism is shown on Fig. 1. Fig 2 and Fig 3 represents the schematics and weighted graph of the sewing machine mechanism respectively. In this mechanism link 2 is crank and coupled with the drive wheel and the thread puller.

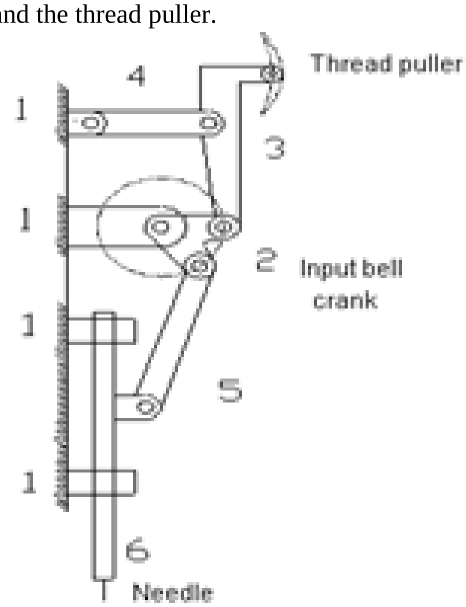

Fig. 1 Physical representation

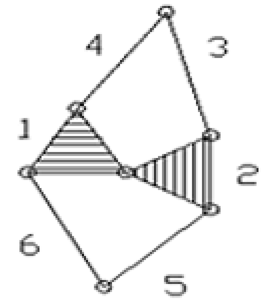

Fig. 2 Schematic representation

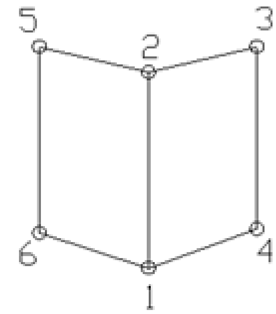

Fig. 3 Weighted graph

3.1 Matrix representation of sewing machine

It is usual to represent an n-link simple jointed kinematic chain by an n^{th} order systematic zero-one.

Matrix A = a_{ij} in which a_{ij} = 1, if link i is connected to link j through a kinematic pair. Otherwise if i is not connected to j then a_{ii} = 0 for all i.

So the matrix for the six link chain for sewing machine mechanism is [A].

[A] =

0	1	0	1	0	1
1	0	1	0	1	0
0	1	0	1	0	0
1	0	1	0	0	0
0	1	0	0	0	1
1	0	0	0	1	0

This matrix A is also known as adjacency matrix of the kinematic chain and use for determining its structural characteristic polynomial equation.

3.2 Computation for characteristic polynomial

Let the characteristic polynomial of the matrix A be represented by Eq. (1).

$$a_0x^n + a_1x^{n-1} + a_2x^{n-2} \ldots\ldots\ldots\ldots + a_{n-1}x + a_n \quad (1)$$

where, $a_0, a_1, a_2 \ldots\ldots a_n$ are the co-efficients.

Bocher's formula given by Eq. (2).

$a_0=1$

$$a_j = \frac{-1}{j}\sum_{r=1}^{j} a_{j-r} s_{rj} \quad (2)$$

where,

j=1, 2, 3, 4………..n

$S_r = T_r (A^r)$ = Trace of matrix (A)

Thus to calculate the co-efficients of characteristic polynomial we need to compute the power of the matrix A upto A^n and then use the Bocher's formula.

The powers of matrix [A] are A^2, A^3,…………. A^6 from these powers of the matrix the traces are obtained as

$S_1 = 0 \quad = \quad Tr\,(A)$
$S_2 = 14 \quad = \quad Tr\,(A^2)$
$S_3 = 0 \quad = \quad Tr\,(A^3)$
$S_4 = 70 \quad = \quad Tr\,(A^4)$
$S_5 = 0 \quad = \quad Tr\,(A^5)$
$S_6 = 398 = \quad Tr\,(A^6)$

Now using the Bocher's formula

$a_0=1$
$a_1=s_1=0$
$a_2= -1/2(a_1\, s_{1+}\, s_2) =-7$
$a_3= -1/3(a_2\, s_{1+}\, a_1\, s_{2+}\, s_3) =0$
$a_4= -1/4(a_3 s_{1+}\, a_2\, s_{2+}\, a_1\, s_3+ s_4) =7$
$a_5= -1/5(a_4\, s_{1+}\, a_3\, s_{2+}\, a_2\, s_3+ a_1\, s_4+ s_5) =0$
$a_6= -1/6(a_5\, s_{1+}\, a_4\, s_{2+}\, a_3\, s_{3+}\, a_2\, s_4+ a_1\, s_5+ s_6) =-1$

So, the characteristic polynomial for sewing machine chain is given by Eq. (4)

$$X^6 - 7X^4 + 7X^2 - 1 \quad (4)$$

4. Isomorphism

Two chains are isomorphic only if their characteristic polynomial equations are same. This method is explained with the help of six link single degree of freedom kinematic chain of sewing machine. Let us test whether this chain is isomorphic with six links for brake drum and straight line motion mechanisms.

4.1 Brake drum mechanism

Fig. 4 represents brake drum mechanism. Fig. 5 and Fig. 6 represent the schematics and weighted graph of the brake drum mechanism respectively.

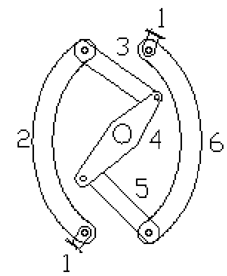

Fig. 4 Physical representation

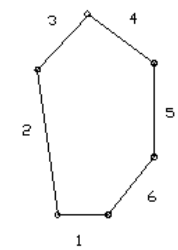

Fig. 5 Schematic representation

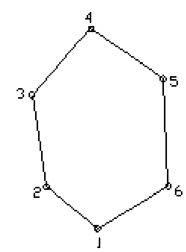

Fig. 6 Weighted graph

The matrix for brake drum mechanism is obtained from the tree graph.

Matrix for brake drum mechanism:-

$$[A] = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Thus to calculate the co-efficients of characteristic polynomial we need only compute the power of the matrix A upto A^n and then use the Bocher's formula.

Traces and characteristic polynomial co-efficients are:-

$S_1 = 0$
$S_2 = 12$
$S_3 = 0$
$S_4 = 136$
$S_5 = 0$

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$S_6 = 132$

Now using the Bocher's formula

$a_0 = 1$
$a_1 = s_1 = 0$
$a_2 = -1/2(a_1 s_1 + s_2) = -6$
$a_3 = -1/3(a_2 s_1 + a_1 s_2 + s_3) = 0$
$a_4 = -1/4(a_3 s_1 + a_2 s_2 + a_1 s_3 + s_4) = 9$
$a_5 = -1/5(a_4 s_1 + a_3 s_2 + a_2 s_3 + a_1 s_4 + s_5) = 0$
$a_6 = -1/6(a_5 s_1 + a_4 s_2 + a_3 s_3 + a_2 s_4 + a_1 s_5 + s_6) = -4$

So the characteristic polynomial for brake drum chain is given by Eq. (5).

$$X^6 - 6X^4 + 9X^2 - 4 \tag{5}$$

4.2 For Straight line motion mechanism

Fig. 7 represents brake drum mechanism. Fig. 8 and Fig. 9 represent the schematics and weighted graph of the straight line motion chain.

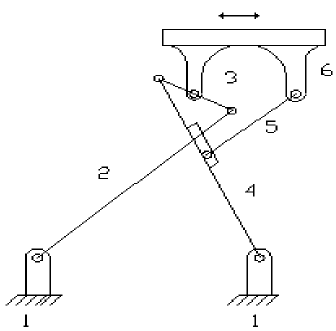

Fig. 7 Physical representation

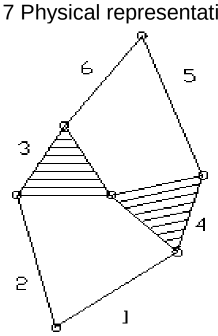

Fig. 8 Schematic representation

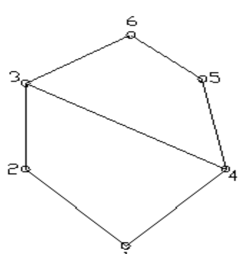

Fig. 9 Weighted graph

The matrix for straight line motion chain is obtained from the tree graph.
Matrix for straight line motion chain:-

$$[A] = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

Traces and characteristic polynomial co-efficients are:-

$S_1 = 0$
$S_2 = 14$
$S_3 = 0$
$S_4 = 70$
$S_5 = 0$
$S_6 = 398$

Now using the Bocher's formula

$a_0 = 1$
$a_1 = s_1 = 0$
$a_2 = -1/2(a_1 s_1 + s_2) = -7$
$a_3 = -1/3(a_2 s_1 + a_1 s_2 + s_3) = 0$
$a_4 = -1/4(a_3 s_1 + a_2 s_2 + a_1 s_3 + s_4) = 7$
$a_5 = -1/5(a_4 s_1 + a_3 s_2 + a_2 s_3 + a_1 s_4 + s_5) = 0$
$a_6 = -1/6(a_5 s_1 + a_4 s_2 + a_3 s_3 + a_2 s_4 + a_1 s_5 + s_6) = -1$

So the characteristic polynomial for straight line motion chain is given by Eq. (6).

$$X^6 - 7X^4 + 7X^2 - 1 \tag{6}$$

Thus, from Eq. (4), (5) and (6), it clearly shows that the Sewing machine chain is not isomorphic with brake drum chain but it is isomorphic with straight line motion chain.

5. Degree of Freedom

Degree of freedom can be determined by the using the Grubbler's criterion given by Eq. (7)

$$F=3(L-1)-2j \tag{7}$$

Where,

L= Number of links
j = Number of joints

$F = 3(6-1)-2x7 = 1$

Thus the degree of freedom of sewing machine chain is one. But the chain can have three types of degree of freedom viz. total, fractionated and partial.

5.1 Test for fractionated degree of freedom

The test for fractionated degree of freedom as proceed as follow:-

1) Check for the number of rows with more than three non-zero entries in them (it is obvious that a separation link has to be at least a quaternary link or the cut vertex must have a degree of at least four). If there is no such row the chain cannot have fractionated degree of freedom. If there is one or more such row, proceed to next step.
2) Obtained matrix A_k [(n-1) x (n-1)] order by deleting the row and column from matrix A (n x n) where k_{th} row has more than three non-zero entries. So that A_k is obtained.
3) Compute what is called the reachability matrix $R_{k.}$ So
   $R_{k\,=}\ I+A_K+A_K^2+A_K^3+A_K^4+A_K^5+A_K^6+\ldots..+A_K^{n-2}$
   Where I is the identity matrix and $A_{K,}$ A_K^2, A_K^3, A_K^4, A_K^5, A_K^6 are the power of A_K. All the computations involved in finding Rk are carried out as per Boolean algebra viz. 1+1=1, 1+0=1, 0+1=1, 1x0=0, 0x1=0, 1x1=1.
4) Check for the presence of zeros in $R_{k.}$ The presence of zeros in R_k indicates that k is a cut vertex and that the graph becomes disconnected after deleting the vertex k. This means that the kinematic chain has fractionated degree of freedom with k as a separation link. A non zero ij_{th} entry in R_k indicates that vertex j is reachable from vertex i and vice versa in the graph of A_k via a path (n-2) or fewer edge. If there is no zero entries in R_k proceed to next step.
5) If k is the only row in a with more than three non-zero entries we conclude that there is no cut vertices in the graph and chain does not have fractionated degree of freedom and does not represent fractionated chain. Otherwise we repeat the steps (2) to (4) with other rows consisting of more than three non-zero entries in them. This is done till there are no more such rows in the matrix. If the test is not satisfied for all such rows we conclude that the chain cannot have fractionated degree of freedom. It may have partial or total freedom.

As it is clear from the diagram the sewing machine does not contain any quaternary link, so the mechanism does not have fractionated degree of freedom because presence of quaternary link is necessary for fractionated degree of freedom.

5.2 Test for partial degree of freedom

Sewing machine chain does not have fractionated degree of freedom then test for partial degree of freedom can be carried out. The chain having degree of freedom greater than 2 and 3 cannot have total degree of freedom and hence have fractionated or partial degree of freedom for such chains, if they do not satisfy the test for fractionated degree of freedom, it is conducted that they have partial degree of freedom. The proposed test need to be apply only for chains having degree of freedom greater than 2 and 3. Since the sewing machine chain has degree of freedom one thus it does not have partial degree of freedom.

6. Computer Program

A computer program using MATLAB has been developed to perform the structural analysis of kinematic chains, and to obtain the characteristic equation of the kinematic chain. The flow chart describing the step-by-step procedure is shown in Fig. 6.

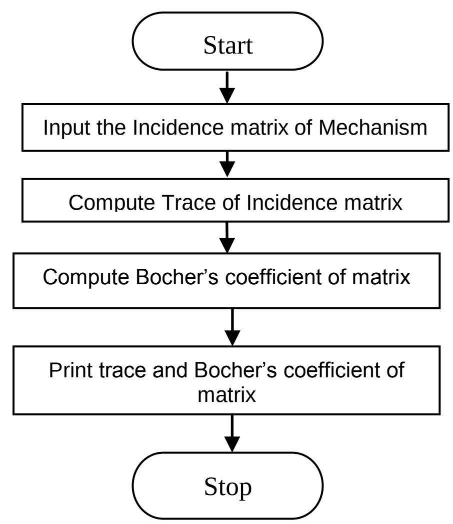

Fig. 6 Flowchart of Algorithm

7. Results and Discussion

7.1 The characteristic polynomial equation from which Sewing machine chain is given as:

$$X^6 - 7\,x^4 + 7\,x^2 - 1$$

7.2 Isomorphism

1. Chain for sewing machine is not isomorphic with Brake drum chain as the characteristic equations derived are not similar. The equation derived for brake drum chain is:

$$X^6 - 6X^4 + 9X^2 - 4$$

2. Chain for sewing machine is isomorphic with straight line motion chain as the characteristic equations derived are similar. The equation derived for straight line motion chain is:

$$X^6 - 7X^4 + 7X^2 - 1$$

7.3 Degree of Freedom

Sewing machine mechanism has total degree of freedom one but does not have fractionated and partial degree of freedom chain.

8. Conclusion

1. Characteristic polynomial equations have been used for structural analysis and determination of isomorphism between two or more kinematic chains. The co-efficients of the characteristic equations represents the strength of the kinematic chains.

2. During the analysis of Sewing machine mechanism it is found that the links two and six are stronger other than the link four because the co-efficient with the link four is negative which shows that the chain is weaker at link four.

3. Sewing machine chain when checked for isomorphism, clearly shows that the sewing machine chain is isomorphic with straight line motion chain.

4. Since, Sewing machine does not contain any quaternary link, so the mechanism does not have fractionated degree of freedom and therefore sewing machine chain cannot be broken into two closed loops.

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Shubham Verma, M.Tech student, received B.Tech in Mechanical Engineering from Kurukshetra University, Kurukshetra, and presently pursuing M.Tech in Machine Design from National Institute of Technology, Kurukshetra (India).

V. P. Singh, an Associate Professor in Department of Mechanical Engineering, National Institute of Technology, Kurukshetra. He is actively involved in research and teaching. Her research field includes Machine Design, Mechanical Vibration, Industrial Robotics and Graph Theory. He has number of papers published in conference proceedings and journals. He is a member of ISTAM and ISTE.