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Analysis of a Bulk Arrival Bulk Service Queueing Model for Non Reliable Server

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Abstract

In this paper, we study the optimal operation of a single removable and non-reliable server in a Markovian queuing system under steady-state conditions. The system is in idle state before the arrival of customer and after the arrival of customer it is in working state, it may breakdown and systems goes to vacation during that period. Here, there is bulk arrival of customer and bulk service provided to them.

Inter-arrival and service time distributions of customers are assumed to be exponentially distribution. Breakdown and repair time distributions of the server are assumed to be exponentially distributed.

Key word: Inter-arrival, service time, Break down, repair time, Bulk service, Non reliable server

INTRODUCTION

In general the service facility in a queueing system is assumed to provide uninterrupted service to the customer unit. There are however, many real-life situations where the service gets interrupted due to breakdown in service mechanism. Therefore, the breakdown period is the length of time per cycle when the server is turned on found to broken down and customer are waiting for their service. Such type of break down are common in industries, telephone traffic etc. Most of the studies of this type of including those of Gaver (1962) and Thiruvengudan (1963) single server system with exponential distribution. White and Christic (1958) seem to have studied first a queueing model with single channel which provide service with breakdown. The term removable server is just an abbreviation for the system of turning on and turning off the server, depending on number of customer in the system. A non-reliable sever means that a server is typically subject to unpredictable breakdowns. The server is removable under N-Policy; turn the server on whenever N (N >1) or more customer are presents, turn the server off only when no customer are presents. For the non-reliable server, Avi – Itzhak and Naor[1967] studied the ordinary M/M/1 queueing model where the service does not depend upon the number of customer in the queue. Neut and Lucanton [1983] Studied a Markovian queueing system single/multiple server subject to breakdown and repair. In these type of breakdowns the customer are waiting to serve and server goes for vacations and after repairing he come back for service. But here we assume that during the breakdown period the server can do some another work, additional work aside from service of customers arriving at the queue or pending work to utilize that time. The server schedule the service of these tasks during the breakdown period.

Assumptions :

(i) The arrival of customer are according to Poisson process with parameter λ.

(ii) Service time of customer is exponentially distributed with parameter μ.

iii) As long as the server on, it will serve the customer immediately.

(iv) When it is working, it is assumed that breakdown can happen at any time, with poisson rate ϕ_1.

(v) Whenever breakdown occurs it is repaired immediately with repair rate ϕ_2 where repair rate are assumed to be exponentially distributed.

(vi) The server goes on vacation when there is no customer in the server and come back only when N customer in the queue.

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Steady – state equation

i = 0 represents the server is in idle state.

i = 1 represents the server is on and in working condition.

i = 2 represents the server is turned on and found to be broken.

The steady state equation for $p_0(r)$, $p_1(r)$, $p_2(r)$ are

$$\lambda P_0(0) = \mu P_1(1) \quad \ldots(1)$$

$$\lambda \sum_{m=1}^{r} \xi_m P_0(x) = \lambda \sum_{m=1}^{r} \xi_m (r-m) \qquad 1 \le r \le N-1 \quad \ldots(2)$$

$$\left(\sum_{m=1}^{r} \xi_m \lambda + \phi_1 + \sum_{m=1}^{r} \xi_m \mu \right) P_1(1) = \mu \sum_{m=1}^{r-1} \xi_m P_1(2) + \phi_2 P_2(1) \quad \ldots(3)$$

$$\left(\sum_{m=1}^{r} \xi_m \lambda + \phi_1 + \sum_{m=1}^{r} \xi_m \mu \right) P_1(r) = \sum_{m=1}^{r-1} \xi_m \lambda P_1(r-m) + \mu \sum_{m=1}^{r-1} \xi_m P_1(r+1) + \phi_2 P_2(r)$$

$$m+1 \le r \le N-1 \quad \ldots(4)$$

$$\left(\sum_{m=1}^{N-1} \xi_m \lambda + \phi_1 + \sum_{m=1}^{N-1} \xi_m \mu \right) P_1(N) = \lambda \sum_{m=1}^{N-1} \xi_m (N-1) + \lambda \sum_{m=1}^{N-1} \xi_m P_1(N-1)$$

$$+ \mu \sum_{m=1}^{N-1} \xi_m P_1(N+1) + \phi_2 P_2(N) \quad \ldots(5)$$

$$\left(\sum_{m=1}^{N-1} \xi_m \lambda + \phi_1 + \sum_{m=1}^{N-1} \xi_m \mu \right) P_1(r) = \lambda \sum_{m=1}^{N-1} \xi_m P_1(r-1) + \mu \sum_{m=1}^{r} \xi_m P_1(r+1) + \phi_2 P_2(r),$$

$$r \ge N+1 \quad \ldots(6)$$

Let P(z) denote the generating function of the number of customers in the system, thus

$$P(z) = Q_0(z) + Q_1(z) + Q_2(z)$$

$$P(z) = \left\{ \frac{1-z^N}{1-z} + \frac{\sum_{m=1}^{r} \xi_m \lambda z(1-z^N)(\sum_{m=1}^{r} \xi_m \lambda z(1-z^N) - \phi_2 - \phi_1)}{[\sum_{m=1}^{r} \xi_m \lambda z^2 - (\sum_{m=1}^{r} \xi_m \lambda + \mu + \phi_1)z + \sum_{m=1}^{r} \xi_m \mu] \; [\sum_{m=1}^{r} \xi_m \lambda - \lambda - \phi_1] - \phi_2 \phi_1 z} \right\} \quad \ldots(7)$$

L_S The expected number of customers in the system

$$L_S = L_{IL} + L_{ID} + L_{BD}$$

$$L_S = \frac{(N-1)}{2} + \frac{\rho \left(\sum_{m=1}^{r} \xi_m \lambda \phi_2 + \phi_2 \phi_1 + \phi_1^2 \right)}{\phi_1(\phi_1 - P(\phi_2 + \phi_1))} \quad \ldots(8)$$

Here we can say that the expected number of customers in the system for the ordinary M/M/1 queueing system with non-reliable server is obtained by setting N = 1 in (8).

Determining the Optimal Policy

The optimal value of N is approximately given by

$$R^* \approx \sqrt{\frac{2\lambda(C_s + C_d)[\phi_1 - \rho(\phi_1 + \phi_2)]}{C_h \phi_1}} \qquad \ldots(9)$$

Table 1

Values of expected Mean Queue length with respect to λ

	μ = 1, λ = 5	μ = 1, λ = 7	μ = 1, λ = 9	μ = 1, λ = 10
F(R)	**Ls**	**Ls**	**Ls**	**Ls**
1.5	5.29	3.6	3.05	2.9
1.6	3.79	2.78	2.41	2.3
1.7	2.96	2.26	1.99	1.91
1.8	2.43	1.9	1.7	1.64
1.9	2.06	1.65	1.48	1.43
2.0	1.79	1.45	1.31	1.27
2.1	1.59	1.3	1.18	1.14
2.2	1.43	1.17	1.07	1.04
2.3	1.3	1.07	0.98	0.95
2.4	1.19	0.99	0.9	0.88
2.5	1.1	0.91	0.84	0.82
2.6	1.02	0.85	0.78	0.76
2.7	0.95	0.8	0.73	0.71
2.8	0.89	0.75	0.69	0.67
2.9	0.84	0.71	0.65	0.63

Table-2

Values of expected Mean Queue length with respect to λ

	μ = 3, λ = 9	μ = 3, λ = 11	μ = 3, λ = 13	μ = 3, λ = 15
F(R)	**Ls**	**Ls**	**Ls**	**Ls**
5.0	2.80	1.91	1.53	1.32
5.1	2.65	1.83	1.47	1.27
5.2	2.52	1.75	1.41	1.22
5.3	2.41	1.68	1.36	1.18
5.4	2.30	1.62	1.31	1.14
5.5	2.21	1.56	1.26	1.10
5.6	2.12	1.51	1.22	1.06
5.7	2.05	1.46	1.19	1.03

5.8	1.97	1.41	1.15	1.00
5.9	1.91	1.37	1.12	0.97
6.0	1.85	1.33	1.08	0.95
6.1	1.79	1.29	1.05	0.92
6.2	1.73	1.26	1.03	0.98
6.3	1.68	1.22	1.00	0.87
6.4	1.64	1.19	0.98	0.85

Table 3

Total cost and other operational characteristics

C_0=1 C_f=3 C_1=2 C_2=5 C_h=2 N = 0.87

F(R)	N	Ls	Ls	Ls
1.2	0.87	0.13	20.37	53.12
1.3	0.87	0.13	7.42	27.21
1.4	0.87	0.13	4.54	21.45
1.5	0.87	0.13	3.27	18.92
1.6	0.87	0.13	2.56	17.51
1.7	0.87	0.13	2.11	16.58
1.8	0.87	0.13	1.78	15.95
1.9	0.87	0.13	1.55	15.48
2.0	0.87	0.13	1.37	15.12
2.1	0.87	0.13	1.23	14.84
2.2	0.87	0.13	1.11	14.61

Numerical Results and Conclusion

F(R) decreases as λ or ϕ decreases for any cases

(i) R* decreases in λ for any case;

(ii) The optimum value of R, R* decreases as Cost increases.

(iii) F(R*) increases as μ or ϕ_1 decreases .

(iv) The optimum value of R, R* increases in μ for any case.

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