Analysis of Bulk Queueing Model with Non Reliable Server

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Abstract

Here we will study bulk service to customer under optimal operation of a single removable and non-reliable server in Markovian queueing system under steady-state conditions. The decision maker can turn a server on at customer's arrival or off at service completion. Here it is assumed that the server may breakdown only if working and requires repair at repair facility. Inter-arrival and service time distributions of the customers are assumed to be exponentially distributed. Breakdown and repair time distributions of the server are assumed to be exponentially distributed. The following cost structure is incurred to be system; a holding cost for each customer in the system per unit time, cost per unit time when a server fails, and fixed costs for turning the server on or off. The expected cost function per unit time is developed to obtain the optimal operating policy at minimum cost.

Key words: Non reliable Server, Holding Cost, Fixed Cost, Inter arrival & service time, Minimum cost.

INTRODUCTION

In ordinary queueing problems it is assumed that customers arrive singly at a service facility. However, this assumption is violated in many real-world queueing situations. Ships arriving at a port in convoy, people going to a theater and so on, are some of the examples of queueing situations in which customers do not arrive singly, but in bulk. Also, the size of an arriving group may be a random variable or a fixed number. Mathematically and also from practical point of view, the cases when the size of an arriving group is a random variable, are more common, and also more difficult to handle.

The study of bulk arrival queues may be said to have begun with Erlang's solution of the M/EK/1 queue by Brockmeyer (1948). The major contributions on bulk arrival queues are made by many researchers. Baily considered asituation where service can be done in a fixed batch, Medhi (1984) discussed the idea of the batch size (fixed batch size or variable batch) it depends upon the capacity of server.

We now turn to queueing situations in which arrivals occur singly, but service is in bulk. Bulk service queues have potential applications in many areas e.g. in loading and unloading of cargoes at a seaport, in traffic signal systems, in computer networks where jobs are processed in batches, manufacturing/production systems, in transportation processes involving buses, airplanes, trains, ships and so on. Prem Chand (1988) obtained the explicit time dependent probabilities of given number of arrivals and departures by a given time of a FCFS, single server queueing model in which arrivals are occurring singly and are served in batches of variable sizes.

A bulk queue model is described in terms of inter-arrival times of groups of customers, group sizes, service times of customer batches and batch sizes. A bulk queue in its full generality is extremely complicated even in the single-server case. In batch-arrival batch-service queueing systems, customers arrive in batches, and customers in each batch are served simultaneously. Such types of queueing system provide practical models for performance evaluation in computer and communication signals, e.g., multiprogramming computer systems, for which each program requires the loading of memory units from a main memory store; a circuit-switched telecommunication system which supports a variety of traffic type such as voice, video etc., each of which having different bandwidth requirements and holding-time distributions.

Furthermore, in earlier studies on queues, the service is provided instantaneously. But in many queueing systems occurring in real life situations, particularly where the service facility consists of mechanically operated device, the service gets interrupted due to the occurrence of occasional random failures of the service device. The situations where such kinds of failures are common may be encountered in computer and communication systems, maintenance in production systems etc. In manually operated service facilities the interruptions in service may be on account of strain of service, untimely call by the boss or for similar other reasons. Due to these interruptions in service the various parameters of queueing system are affected. As a result of a breakdown, service facility becomes inoperative and the units demanding service can be served only when it is restored to operative state.

Assumption :-

(i) There are bulk arrival to customer are according to poisson with parameter λ.

(ii) Service times of customers are exponentially distribution with mean $1/\mu$.

(iii) Service provide the service to customer in Bulk.

(iv) Whenever the server is on working state it can go to breakdown with poisson rate $\phi 1$ and go to vacation.

(v) Whenever the server goes for breakdown it repaired at a rate $\phi 2$ where repair times are assumed to be exponentially distributed.

(vi) The server goes on vacation when there is no customer in the server and come back only when N customer in the queue.

Notations

i = 0 represents the server turned off.

i = 1 represents the server turned on & in working state.

i = 2 represents the server turned on & found to be in breakdown state.

$p0(r) \rightarrow$ probability that there are r customers in the system when server in not working ;

r = 0, 1, 2,…. N–1

$p1(r) \rightarrow$ probability that there are r customers in the system when server is in working state.

n = 1, 2…………….

$p2(r) \rightarrow$ probability that there are r customer in the system when system is in working state and found to be broken.

Steady-state expressions for $P_0(r)$, $P_1(r)$ and $P_2(r)$ are

$$\lambda P_0(0) = \sum_{m=1}^{n} \xi_m P_1(1) \quad \ldots(1)$$

$$\lambda P_0(r) = \lambda P_0(r-1) \ldots\ldots. 1 \le r \le N-1 \quad \ldots(2)$$

$$(\lambda + \phi_1 + \sum_{m=1}^{n} \xi_m \mu) P_1(1) = \sum_{m=1}^{n} \xi_m \mu P_1(2) + \phi_1 P_2(1) \quad \ldots(3)$$

$$(\lambda+\phi_1 + \sum_{m=1}^{n} \xi_m \mu) P_1(r)=\lambda P_1(r-1)+ \sum_{m=1}^{n} \xi_m \mu P_1(r+1)+\phi_2 P_2(r) \; 2 \le r \le N-1 \quad \ldots(4)$$

$$(\lambda +\phi_1+ \sum_{m=1}^{n} \xi_m \mu) P_1(N)= \lambda P_0(N-1)+ \lambda P_1(N-1)+ \sum_{m=1}^{n} \xi_m \mu P_1(N+1)+ \phi_2 P_2(N) \quad \ldots(5)$$

$$(\lambda +\phi_1 + \sum_{m=1}^{n} \xi_m \mu) P_1(r) = \lambda P_1(r-1) + \sum_{m=1}^{n} \xi_m \mu P_1(r+1) + \phi_2 P_2(r) \; r \ge N +1 \quad \ldots(6)$$

Let Q(z) denote the generating function of number of customer in the system.

$Q(z) = Q_0(z) + Q_1(z) + Q_2(z)$

$$= \left[\frac{1-z^N}{1-z} + \frac{\lambda z(1-z^N)(\lambda z-\lambda-\phi_1-\phi_2)}{[\lambda z^2-(\lambda+\phi_1+\sum_{m=1}^{n}\xi_m\mu]z+\sum_{m=1}^{n}\xi_m\mu][\lambda z-\lambda-\phi_2]} \right] \quad \ldots(7)$$

Operational Characteristics :- To find the operational characteristic let us suppose E_1, no of customer in the idle state, E_2 and E_3 be no of customer in working state and in broken state respectively. E_s be the number of customer in the system .

$$E_1 = \frac{N-1}{2} - \frac{\lambda(N-1)(\phi_2+\phi_1)}{\sum_{m=1}^{n}\xi_m \mu . 2\phi_2} \quad \ldots(8)$$

$$E_2 = \frac{\lambda}{\sum_{m=1}^{n} \xi_m \mu} \frac{N+1}{2} + \frac{\lambda^2}{\sum_{m=1}^{n} \xi_m \mu^2} \frac{(\lambda + \phi_1 + \phi_2\phi_1 + \phi_2^2)}{\phi_2 \left[\phi_2 - \frac{\lambda}{\sum_{m=1}^{n} \xi_m \mu} [\phi_1 + \phi_2] \right]} \quad \ldots(9)$$

$$E_3 = \frac{\lambda}{\sum_{m=1}^{n} \xi_m \mu} \frac{\phi_2(N+1)}{2\phi_2} - \frac{\lambda^2}{\sum_{m=1}^{n} \xi_m \mu^2} \frac{\phi_1[\lambda - \phi_2 - \phi_1 - \sum_{m=1}^{n} \xi_m \mu]}{\phi_2 \left[\phi_2 - \frac{\lambda}{\sum_{m=1}^{n} \xi_m \mu} [\phi_2 + \phi_1] \right]} \quad \ldots(10)$$

Therefore, number of customer in the system is given by

$E_s = E_1 + E_2 + E_3$

$$Es = \frac{N-1}{2} + \frac{\lambda}{\sum_{m=1}^{n} \xi_m \mu} \frac{(\lambda\phi_1 + \phi_2\phi_1 + \phi_2^2)}{\phi_2(\phi_2 - \rho(\phi_2 + \phi_1))} \quad \ldots(11)$$

Optimum N-Policy :- The idle period, the busy period and the breakdown period and the busy cycle are defined as given below.

(i) The idle period denoted by E[IL], length of time when the server is off.

(ii) Busy period to denoted by BP –when system is on and in the working condition and length of time per cycle when system is turned on and in operation.

(iii) During working condition, breakdown occurs. q that period is denoted by BD- is the length of time per cycle when the server is turned on and found to be breakdown and goes vacation. The customers are therefore waiting to serve.

(iv) Busy cycle are denoted by BC, is the length of time from the beginning of the last idle period to the beginning of the next idle period.

The busy cycle is sum of, idle period, busy period of and the breakdown period, we will obtain

E[BC] = E[IL] + E [BP] + E[BB] + E[BC]

using the memory less property for the exponential distribution, the length of idle period is the sum of N exponential random values each having mean 1/λ. Thus expected length of idle period is given by

$$E[IL] = \frac{N}{\lambda}$$

Table 1

Es_1	Es_2	Es	N	Λ	ϕ_1	ϕ_2	μ	P
2.405	4.5	7.6675	5	0.1	0.5	1	0.5	0.2
2.33125	3.8425	5.91406	5	0.25	0.5	1	0.5	0.5
1.745	2.8956	3.7725	5	0.3	0.5	1	0.5	0.6
1.167	1.45	1.9917	5	0.35	0.5	1	0.5	0.7
0.0823	0.38	0.6101	5	0.21	0.5	1	1	0.8
0.0123	0.12	0.2873	5	0.16	0.5	1	1	0.9
1.505	5.5	12.7775	5	0.1	0.5	1	0.7	0.14
1.53125	3.25	9.47656	5	0.25	0.5	1	0.7	0.36
1.545	2.5	8.3625	5	0.3	0.5	1	0.7	0.43
1.56125	1.75	7.23219	5	0.35	0.5	1	0.7	0.5
1.60125	1.58	6.0231	5	0.45	0.5	1	1	0.45
1.69845	1.31	5.102	5	0.63	0.5	1	1	0.63

Table 2

Es	ϕ_1	ϕ_2	P	λ	μ
4	0.2	0.3	0.5	0.1	0.2
1	0.2	0.3	0.333333	0.1	0.3
0.571429	0.2	0.3	0.25	0.1	0.4
0.4	0.2	0.3	0.2	0.1	0.5
0.307692	0.2	0.3	0.166667	0.1	0.6
0.25	0.2	0.3	0.142857	0.1	0.7
0.210526	0.2	0.3	0.125	0.1	0.8
0.2	0.3	0.5	0.2	0.4	0.2
0.2	0.3	0.4	0.2	0.5	0.2
0.2	0.3	0.333333	0.2	0.6	0.2
0.2	0.3	0.285714	0.2	0.7	0.2
0.2	0.3	0.2	0.8	0.2	
0.2	0.3	0.222222	0.2	0.9	0.2
0.2	0.3	0.2	0.2	1	0.2
4.333333	0.2	0.3	0.5	0.15	0.3
1.444444	0.2	0.3	0.375	0.15	0.4
0.866667	0.2	0.3	0.3	0.15	0.5
0.619048	0.2	0.3	0.25	0.15	0.6
0.481481	0.2	0.3	0.214286	0.15	0.7
0.393939	0.2	0.3	0.1875	0.15	0.8
0.333333	0.2	0.3	0.166667	0.15	0.9

Numerical Result

The table (1) and (2) shows that if service rate increases than queue length decreases and if arrival rate increases than queue length also increases.

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